Time-Frequency Analysis in Visual Signal

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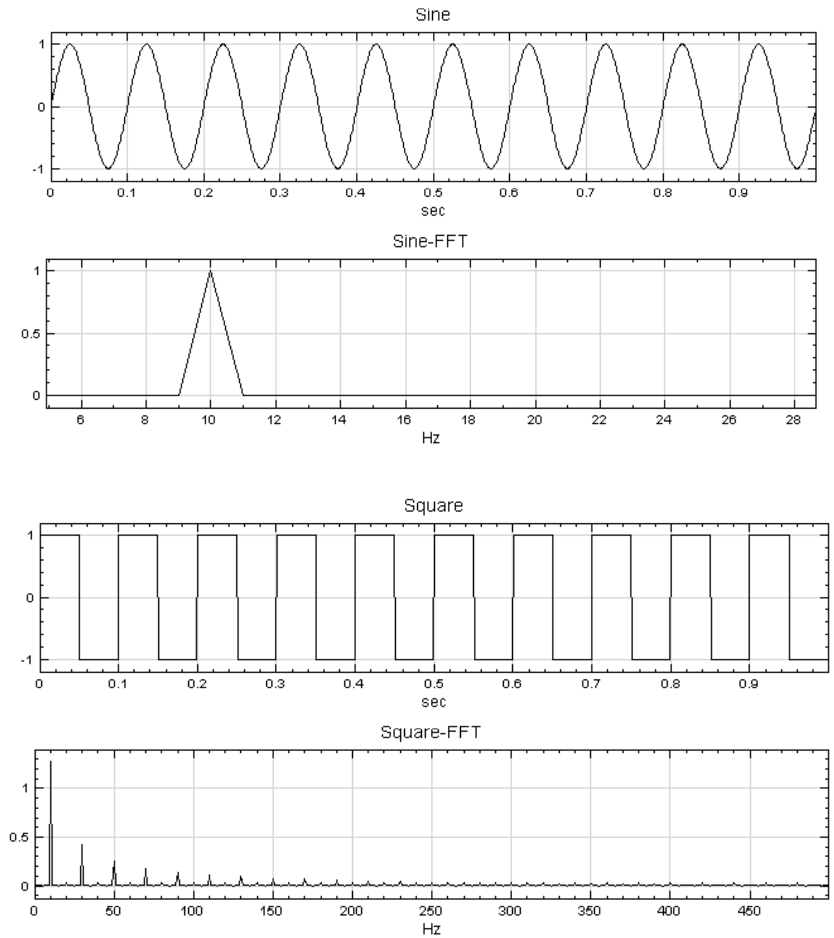
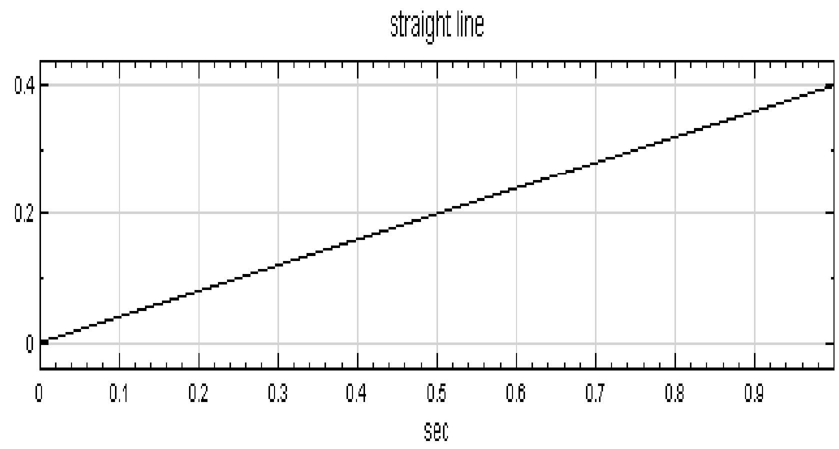
Single frequency?

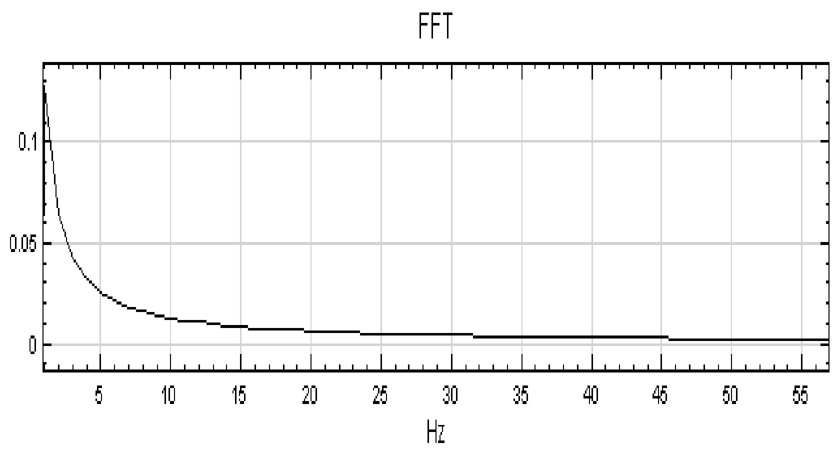
- Is the squared periodical wave of single frequency?
- With Fourier glasses, sine wave is of single frequency while squared wave is of multiple frequencies.
- What is the definition of frequency?
- What do we want to see?



Non-Periodical Signal of many frequencies?

- Does a straight has a frequency?
- Putting on Fourier glasses, we see so many frequencies from a straight line.
- Again what do we want to see?





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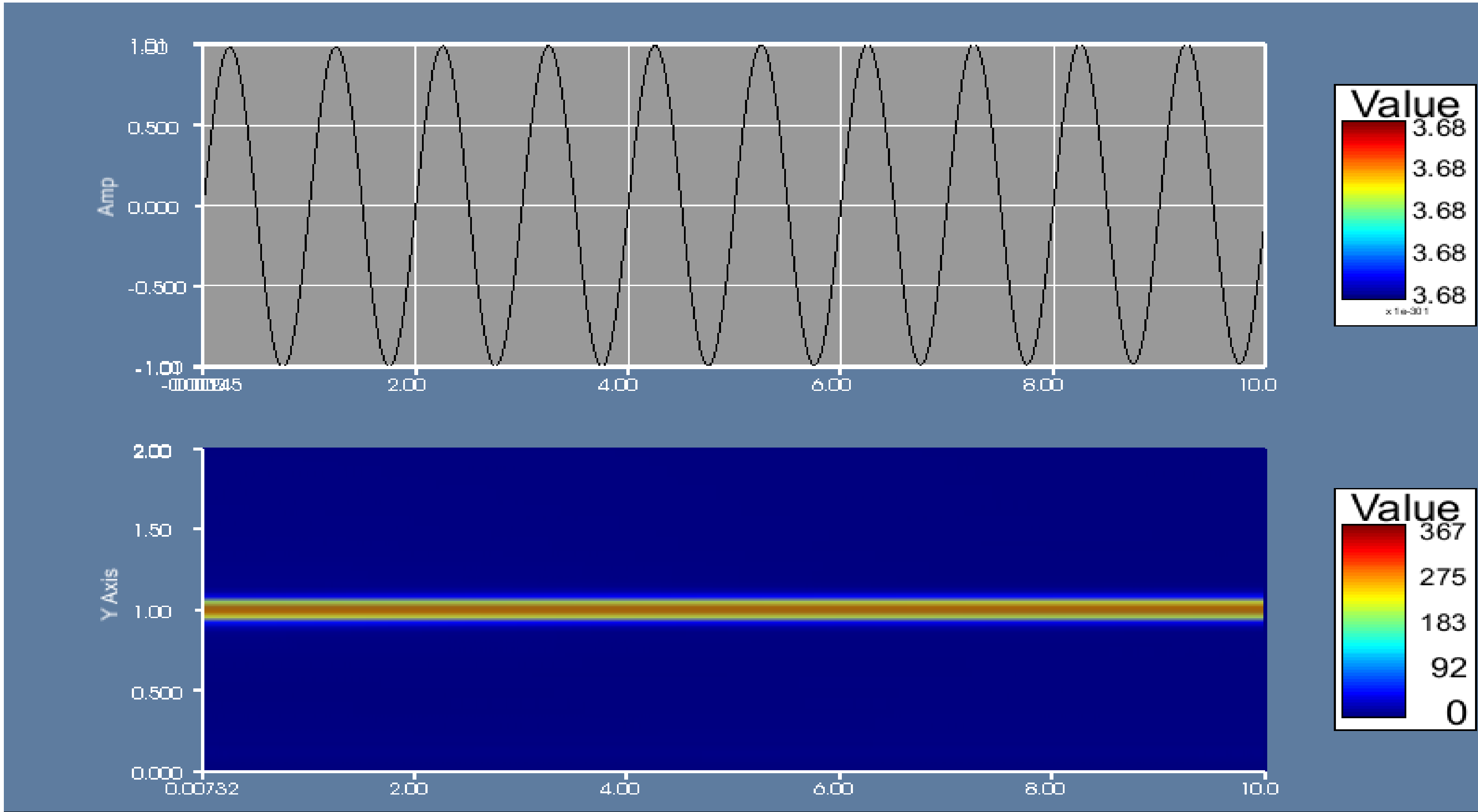
What do we want? Data perception.

- Signal normally is composed of non-periodical part, periodical part, noise, and jump/discontinuity. For data perception, we want to be able to separate all of them. And non-periodical signal is represented in time series plot, noise eliminated, jump/discontinuity taken cared without introducing undesired alias, and periodical signal shown in **time-frequency plot.**

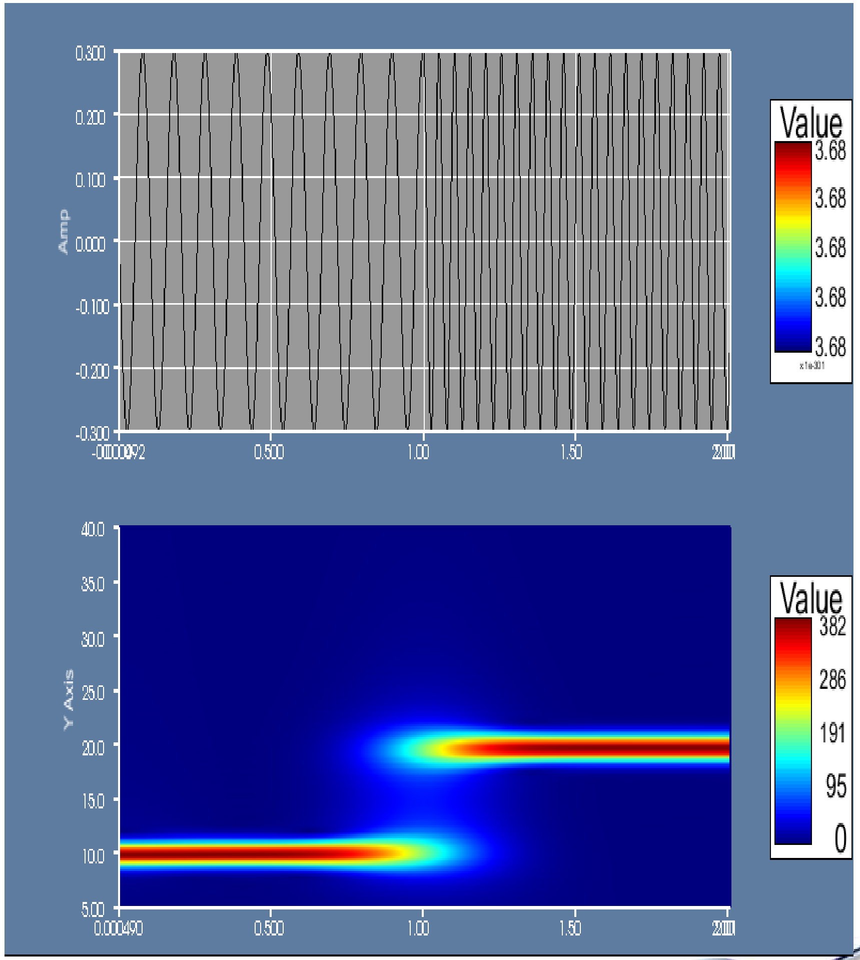
TF Plot: Single frequency



TF Plot: Change of frequency

- Signal with abrupt change of frequency.

$$x(t) = \begin{cases} 0.30\cos(2\times 10\pi t) & ,0 \le t < 1 \\ 0.30\cos(2\times 20\pi t) & ,1 \le t < 2 \end{cases}$$



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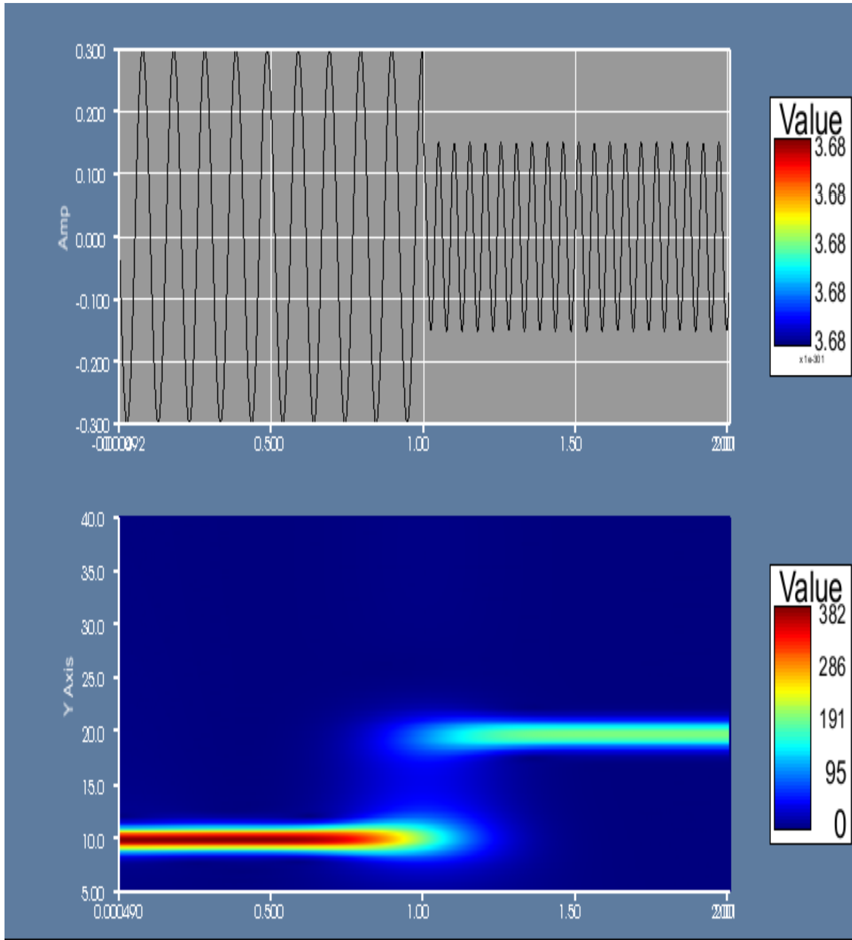
TF Plot: Change of frequency and amplitude

- Signal with abrupt change of frequency and amplitude

$$x(t) = \begin{cases} 0.30\cos(2\times 10\pi t) & ,0 \le t < 1 \\ 0.15\cos(2\times 20\pi t) & ,1 \le t < 2 \end{cases}$$



To be clarified

- What is non-periodical signal?
- **How to remove non-periodical signal?**
- **What is frequency? What is instantaneous frequency?**
- What is noise? How to eliminate noise?
- How to identified jump and discontinuity?
- Signal trace is finite. How to eliminate end effects occurred very often in signal processing?

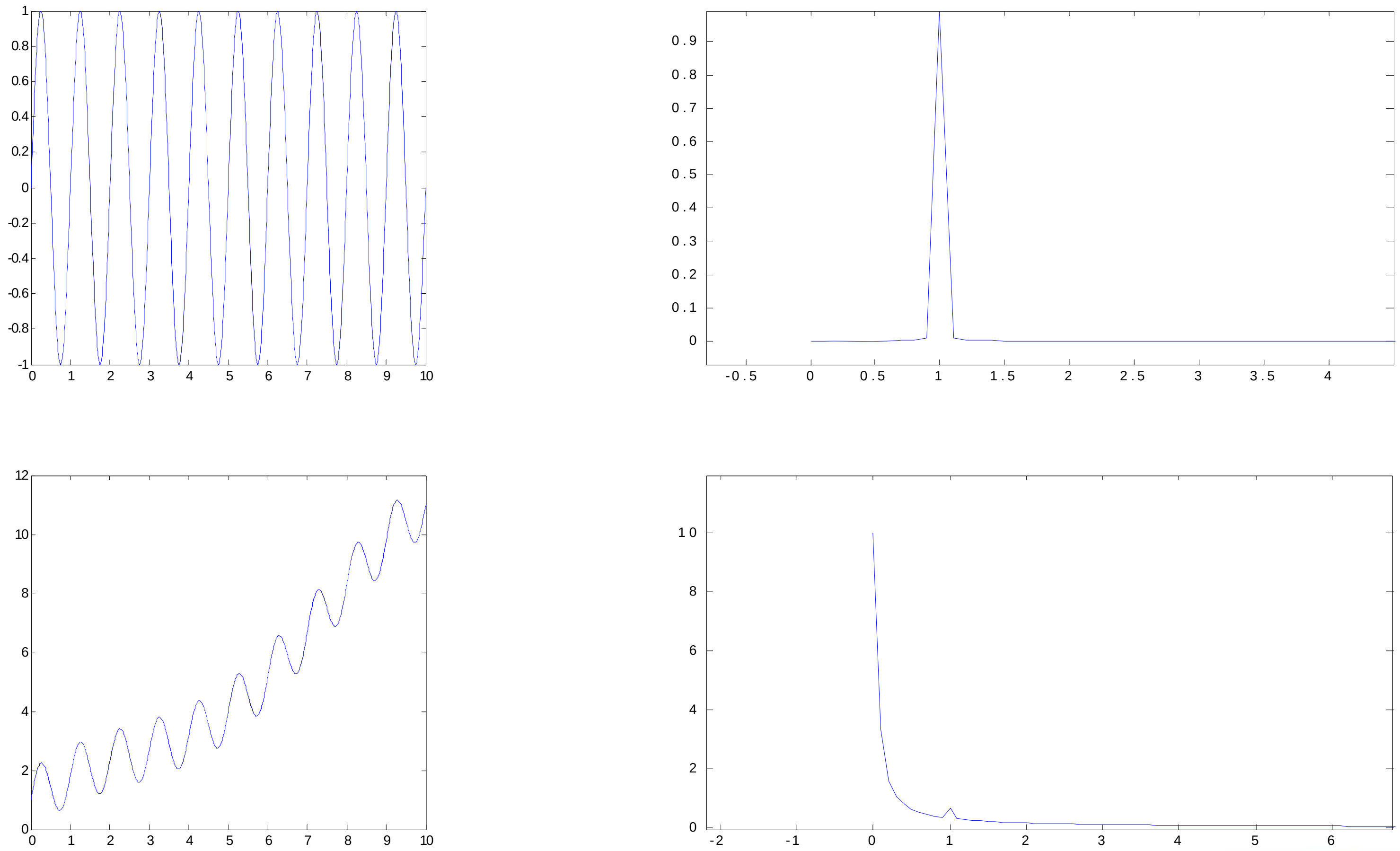
Removal of Non-Periodical Signal

Frequency based filter

Iterative Gaussian Filter

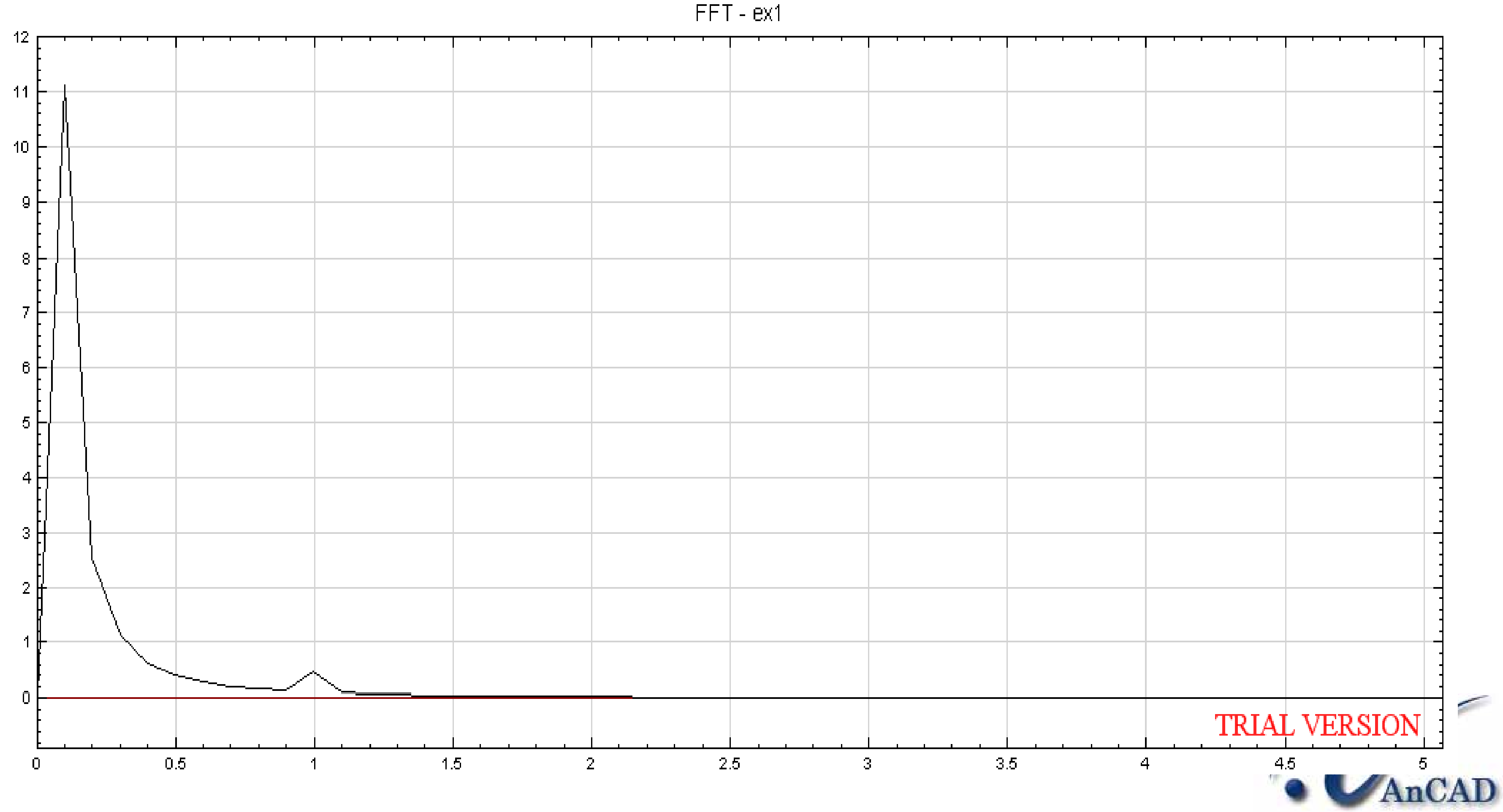
EMD as filter

Containmanation of non-periodical Signal



Removal of Non-Periodical Signal

Frequency based filter



Monotonic Cubic Interpolation

- Avoid Runge effect and end effect
- Uniform or non-uniform over-sampling
- Monotonic condition is slightly released

Spectrum Analyzer

- For better spectrum analysis, the following steps are applied before doing FFT:
- Using Gaussian filter to eliminate non-periodical part
- Removal of signals before and after the first and the last zero crossings via interpolation
- Over sampling the signal via monotonic interpolation that the signal length is a power of 2
- Odd function mapping to ensure the perfect periodic condition up to (N-1)/2-th order of discrete derivatives.

Iterative Gaussian Smoothing Filter

- Removal of Non-Periodical Signal
- Low Pass/High Pass/Band Pass Filter
- Phase Perseverance through any number of iteration steps (theoretically proved) without ripple or artifact in time domain
- Derivative of signal (next version)
- Broken Signal Reconstruction (next version)

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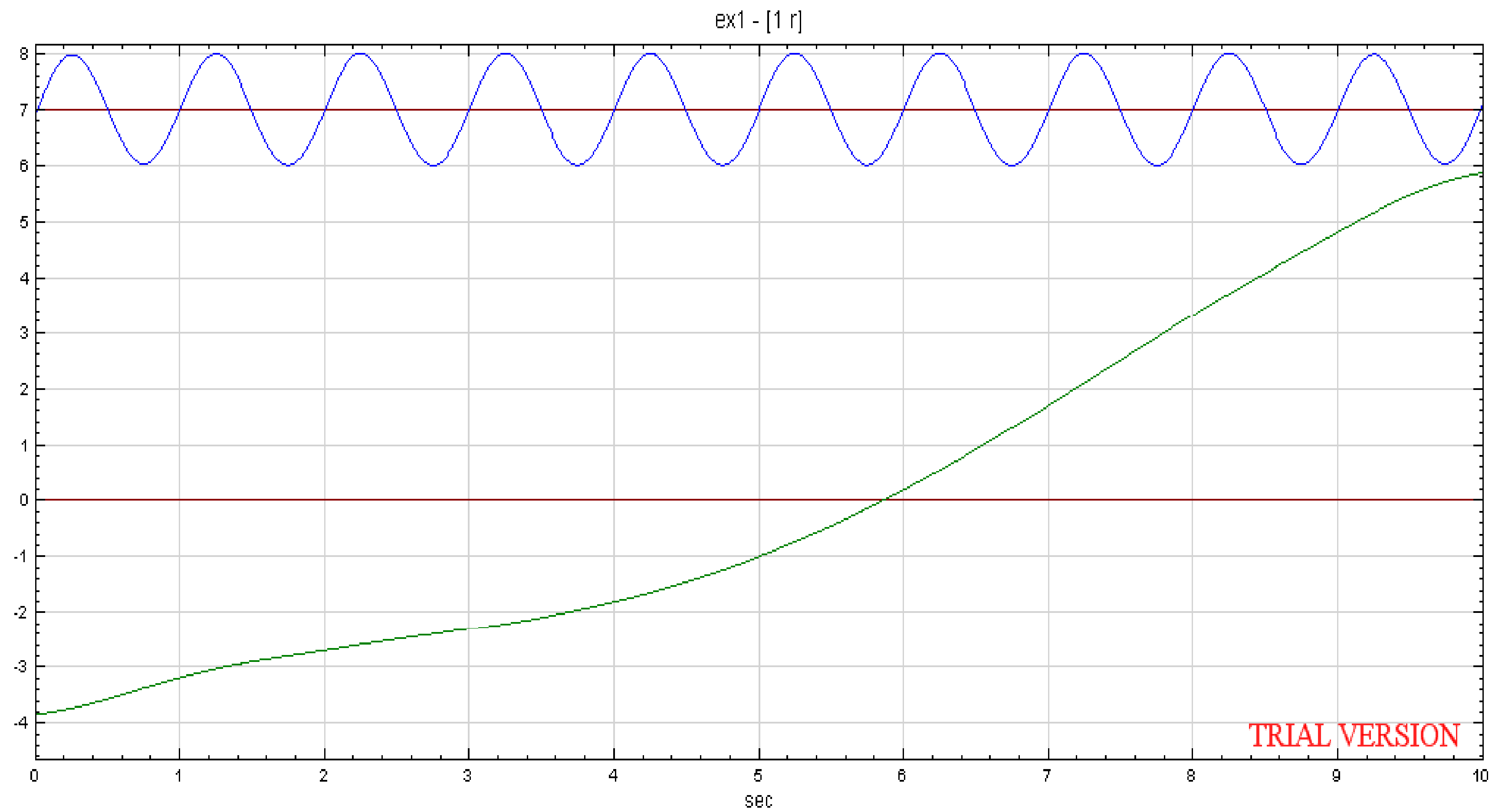
EMD as filter



What is frequency?

The need of instantaneous frequency.

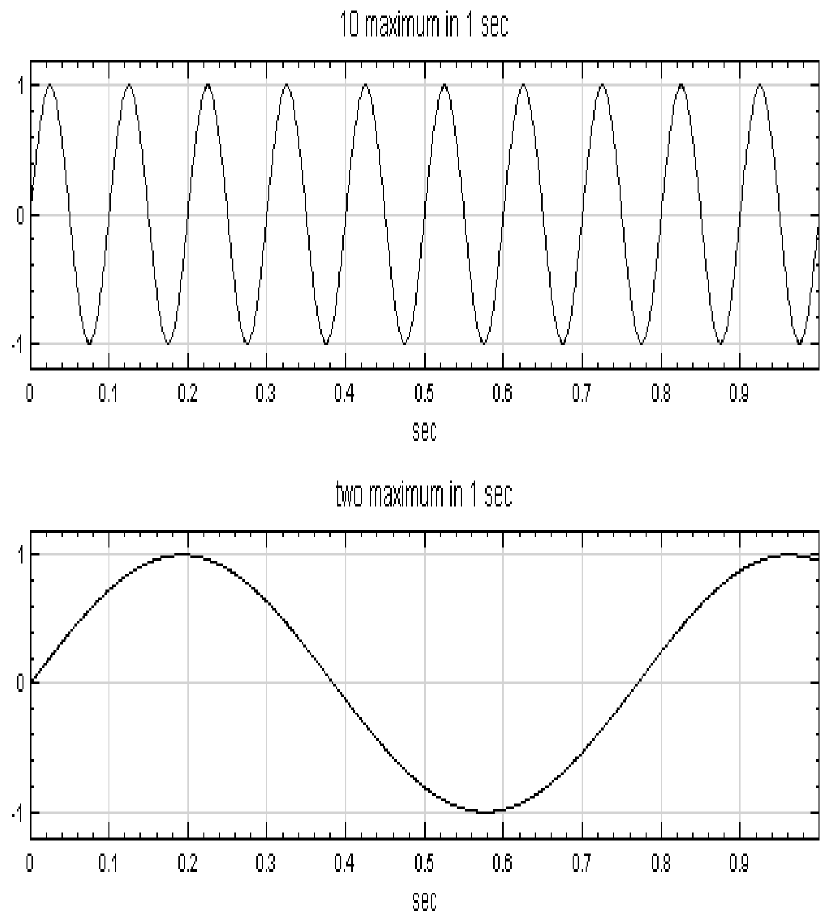
STFT

Morlet Transform

Enhanced Morlet Transform

Frequency definition (1)

- Period is defined as number of events in duration of time. And frequency is the inverse of period.
- The concept of period is an average. So do the frequency.
- Since there is no instantaneous period, there is no instantaneous frequency.
- Putting on Fourier glasses, uncertainty principle comes into play for sinusoidal signal. That is, the lower the frequency to be identified, the longer the duration of signal.



Frequency definition (2)

- In classical wave theory, frequency is defined as time rate change of phase.
- Frequency is an continuous concept. There is need to clarify what we mean by frequency at a certain moment.
- Uncertainty principle does apply.

$$k = \nabla \varphi$$

$$\omega = -\frac{\partial \varphi}{\partial t}$$

$$\frac{\partial k}{\partial t} + \nabla \omega = 0$$

Short-Term Fourier Transform

- Frequency at a certain time is a distribution obtained from Fourier transform. The short period of signal applied to the Fourier transform contains the specific moment of interest.
- In time-frequency analysis, such idea evolves as the Short-Term Fourier Transform (STFT).

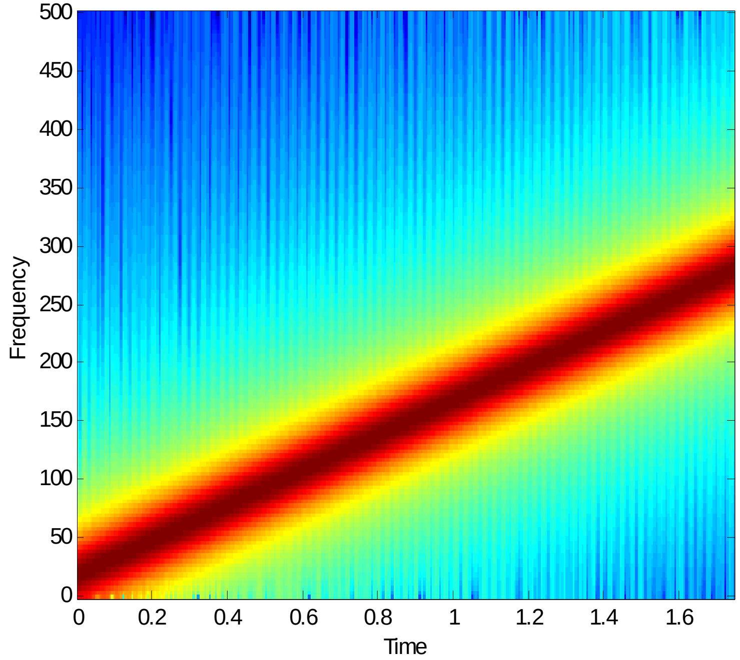
STFT:

$$F(t,\omega) = \int_{-\infty}^{+\infty} f(\tau) g(\tau - t) e^{-i\omega\tau} d\tau$$

Note g is a windowing function.
For Gaussian window, the transform is also known as Gabor Transform.

Gabor transform

- Same chirp signal processed by Gabor transform. (MATLAB)



Challenges in STFT

- Catching low frequency component needs longer time.
- Windowing is needed to avoid end effects.

=> STFT is suitable for band-limited signal like speech and sound.

Morlet Transform

- Scale property in signal is related to frequency property when mother wavelet is Morlet..
- Longer duration of wavelet is used to catch lower frequency component.

$$F(t,s)=\int_{-\infty}^{+\infty} f(\tau)\frac{1}{\sqrt{s}}\psi^{*}(\frac{\tau-t}{s})d\tau$$

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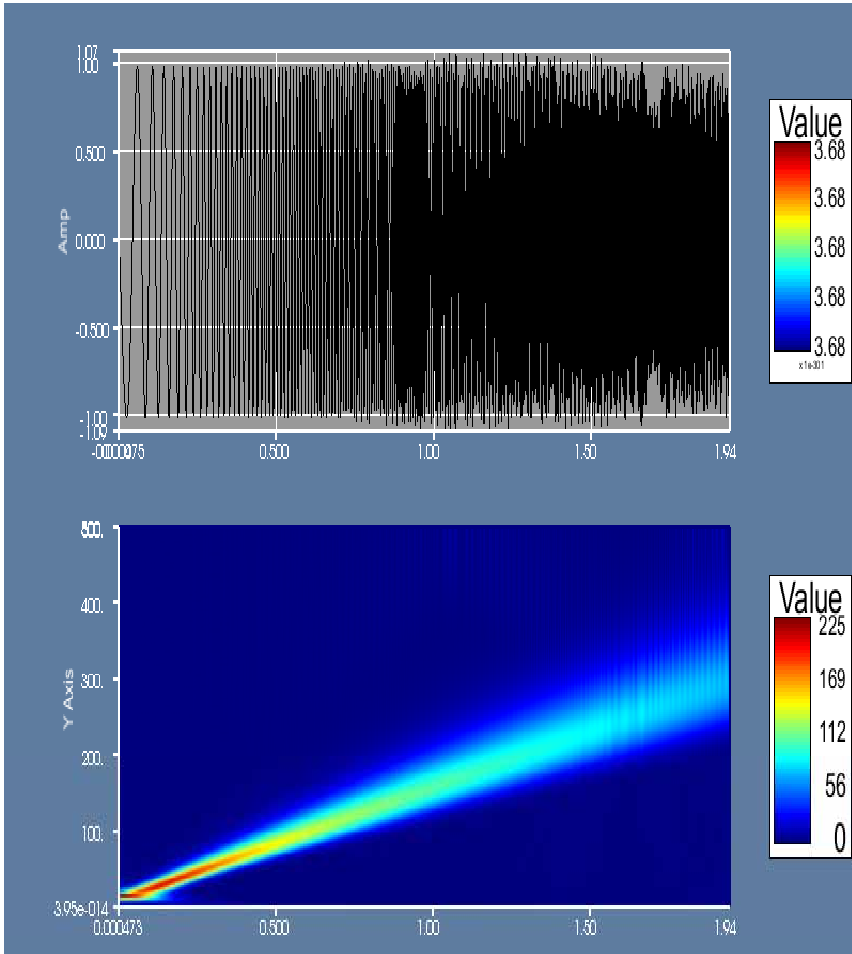
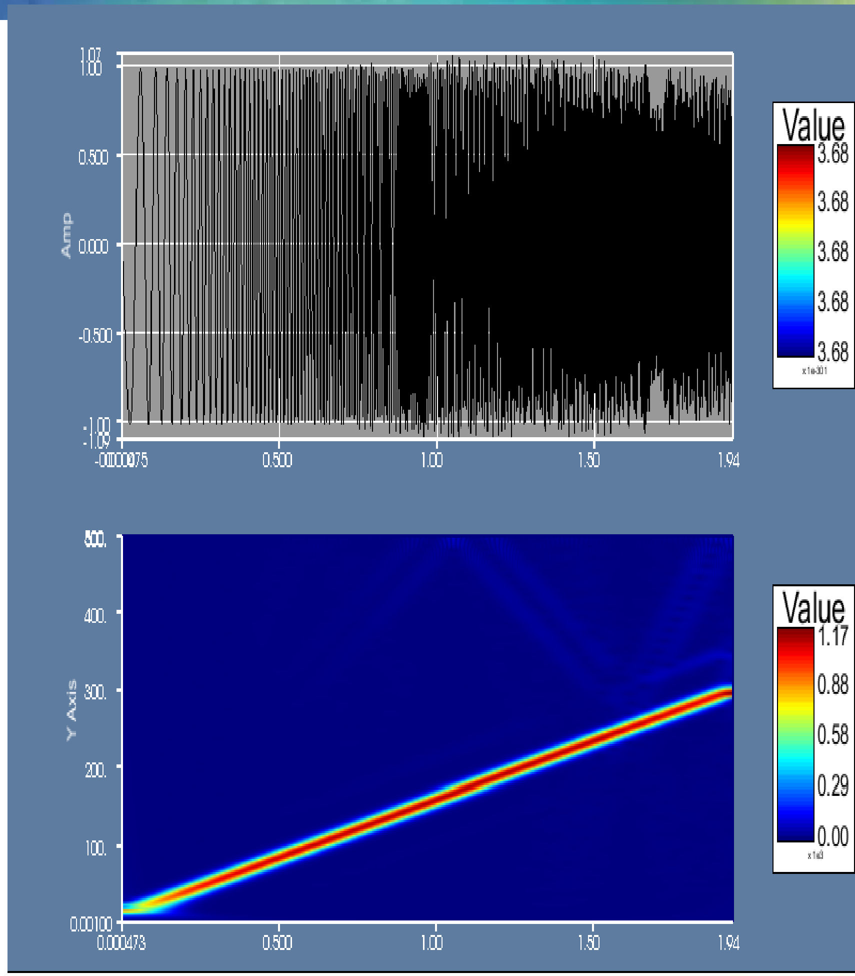
Morlet transform

- Morlet transform on a chirp signal.
- In catching the high frequency spectrum, mother wavelet of short duration of time is used. The spectrum of such wavelet suffers from wide span of frequency, resulting in low resolution, as shown in the right left plot.



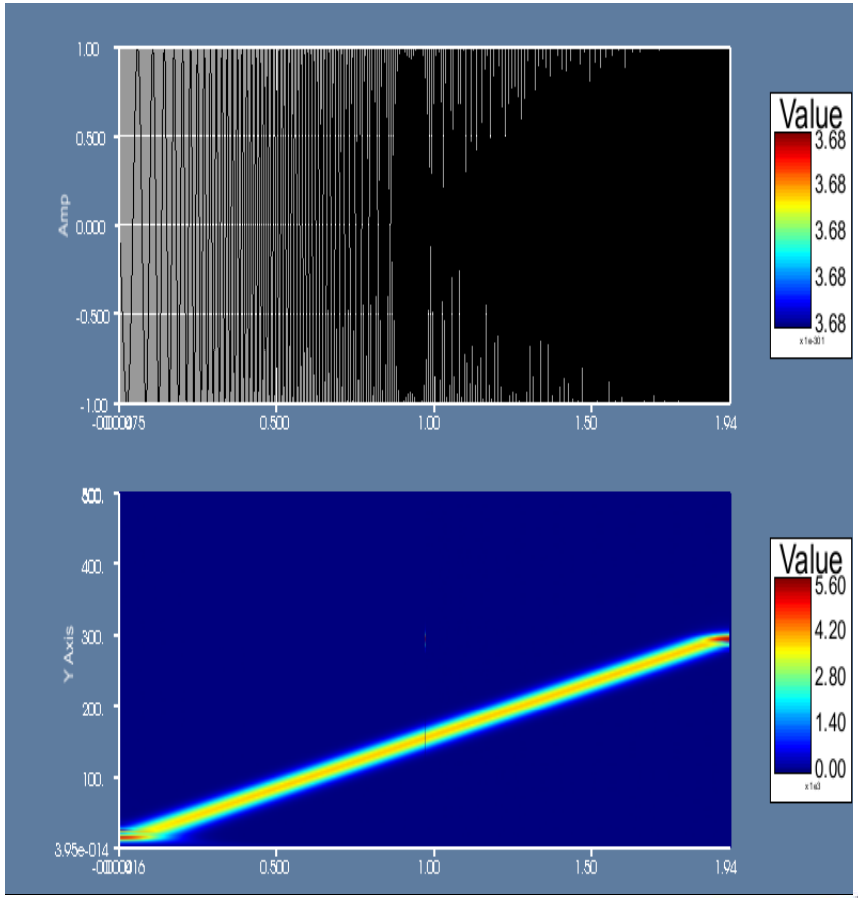
Enhanced Morlet Transform

- By applying Gaussian windowing in frequency domain and knowing that the crossed term of convolution between mother wavelet and signal is the cause of blur, the resolution of Morlet transform can be greatly improved by neglecting the crossed term.
- The fine structure appears in high frequency region is caused by under sampling. The chirp signal is digitized with constant sampling rate.

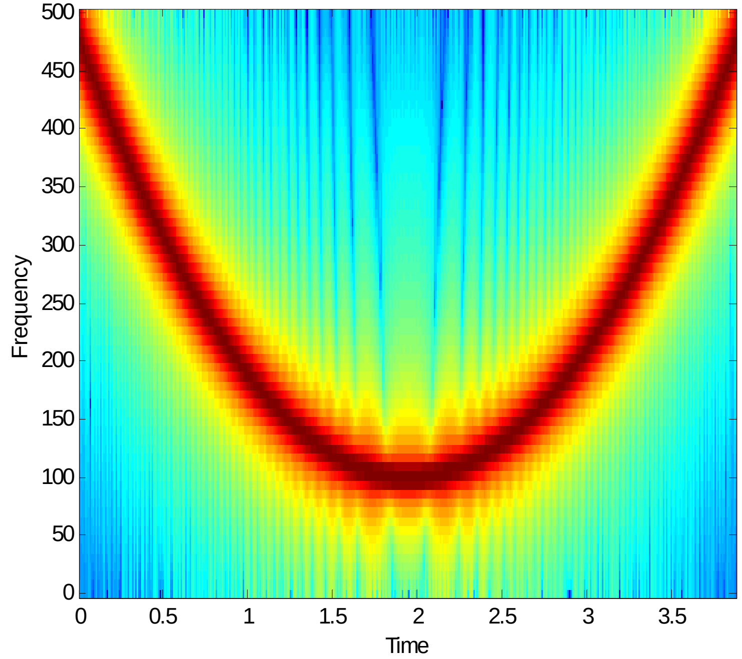


Chirp signal with higher sampling rate

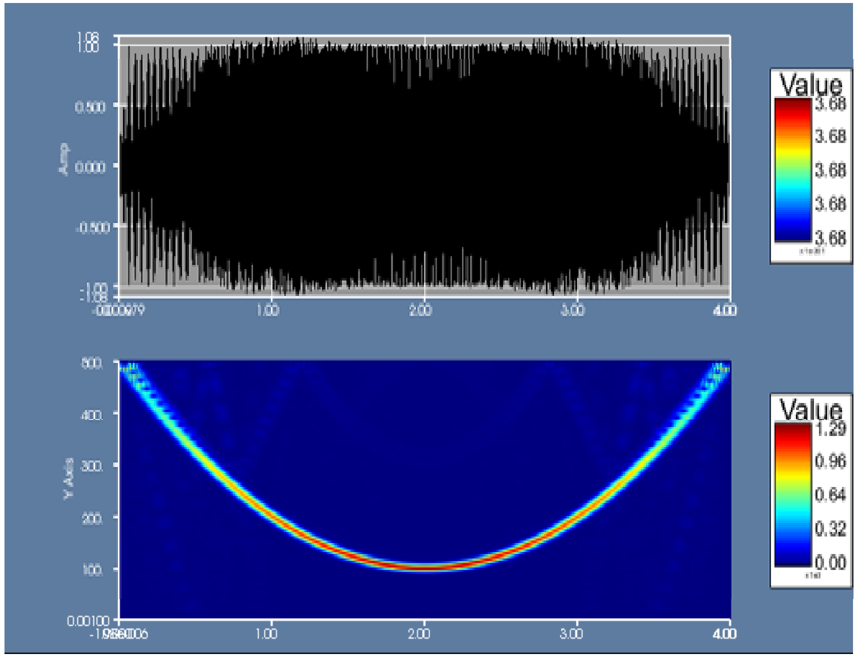
- With higher sampling rate, the fine structure in TF plot disappeared. This is to assure the fine structure pattern is caused by under sampling.



Quadratic Chirp Signal

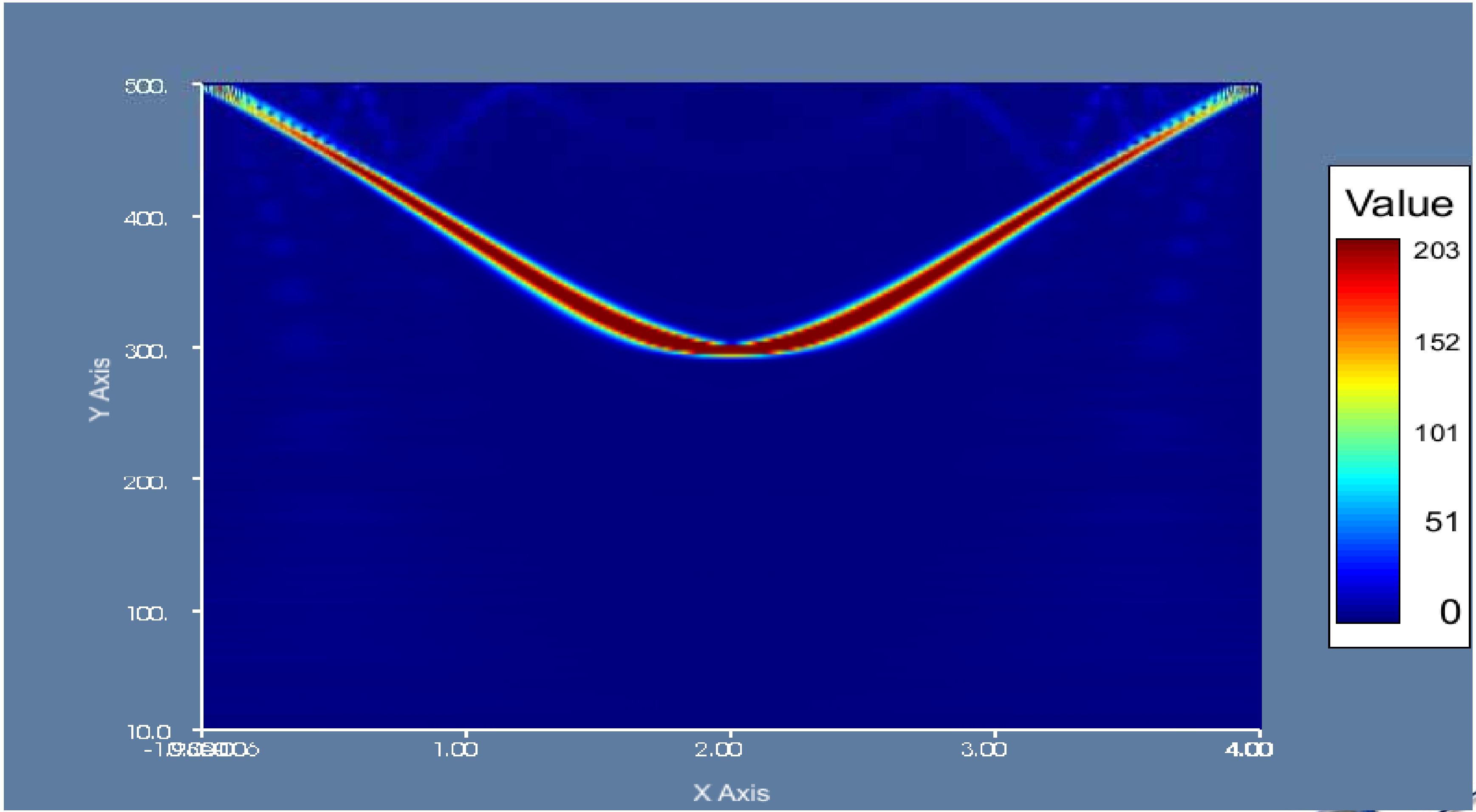


Spectrogram, MATLAB

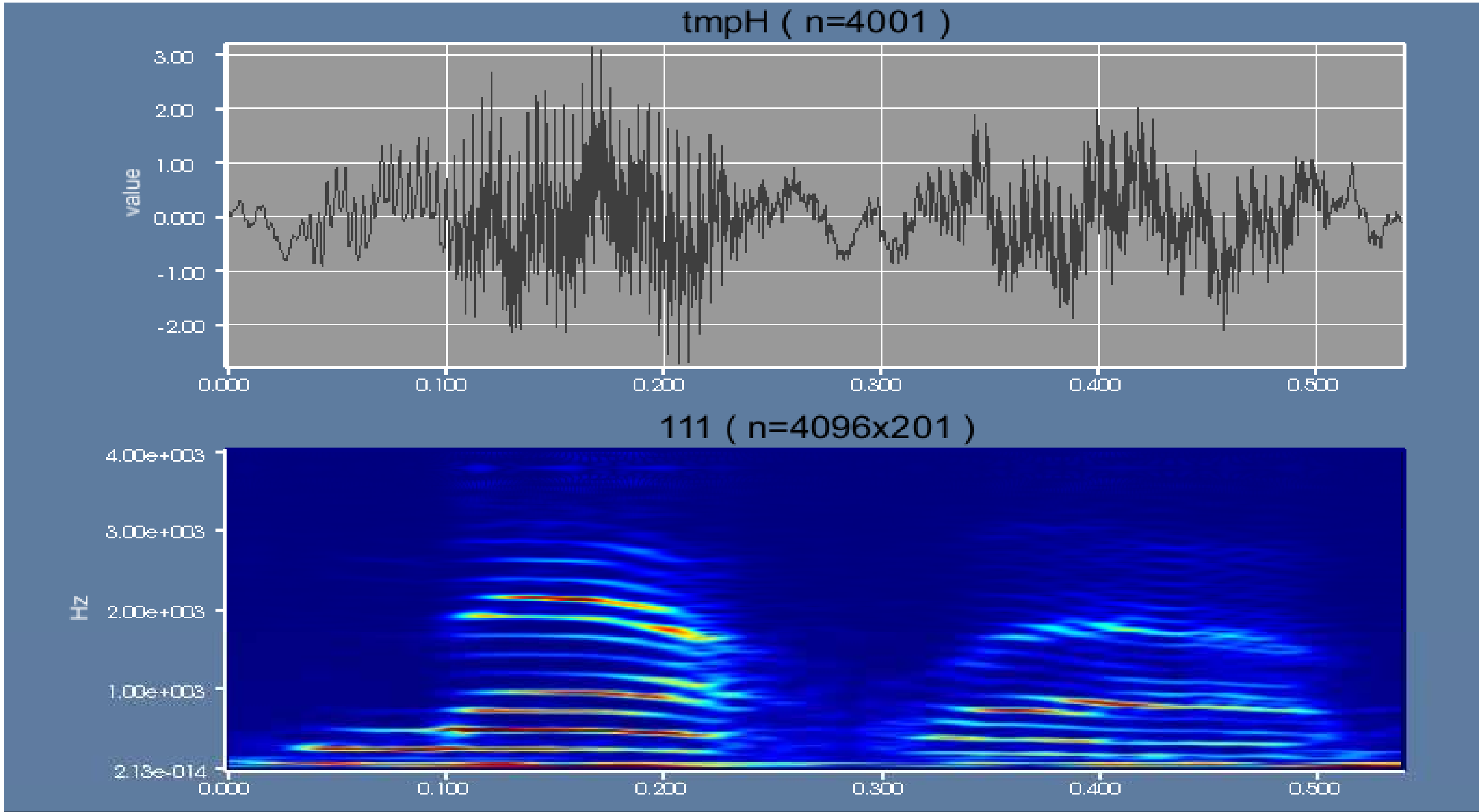


Morlet-Jeng, MATFOR

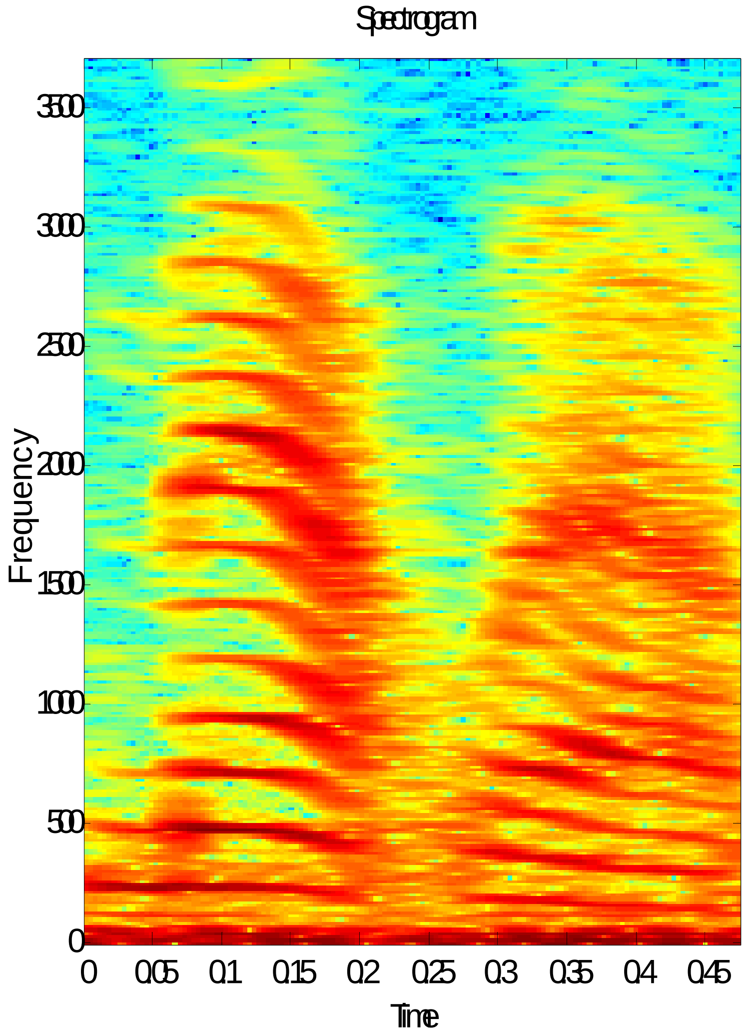
Log unit


500.
400.
300.
200.
100.
10.0
Y Axis
1.00
2.00
3.00
4.00
X Axis
Value
203
152
101
51
0

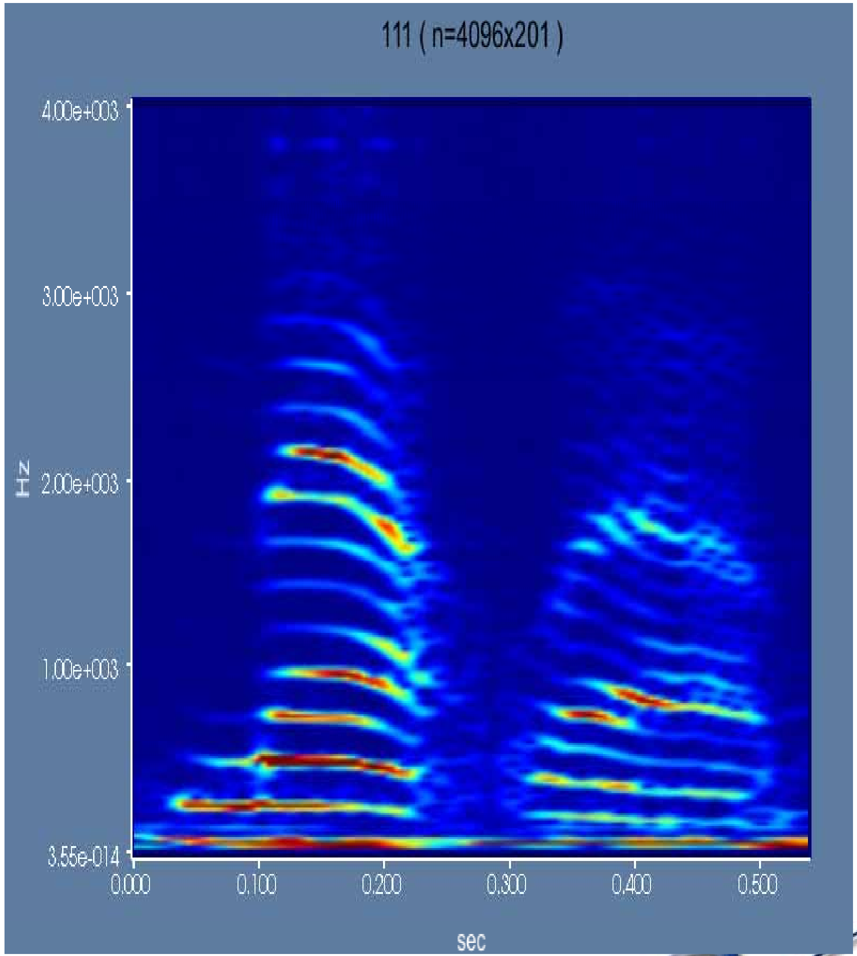
Voice “MATLAB”



Voice “MATLAB”



Spectrogram



Morlet-Jeng Transform

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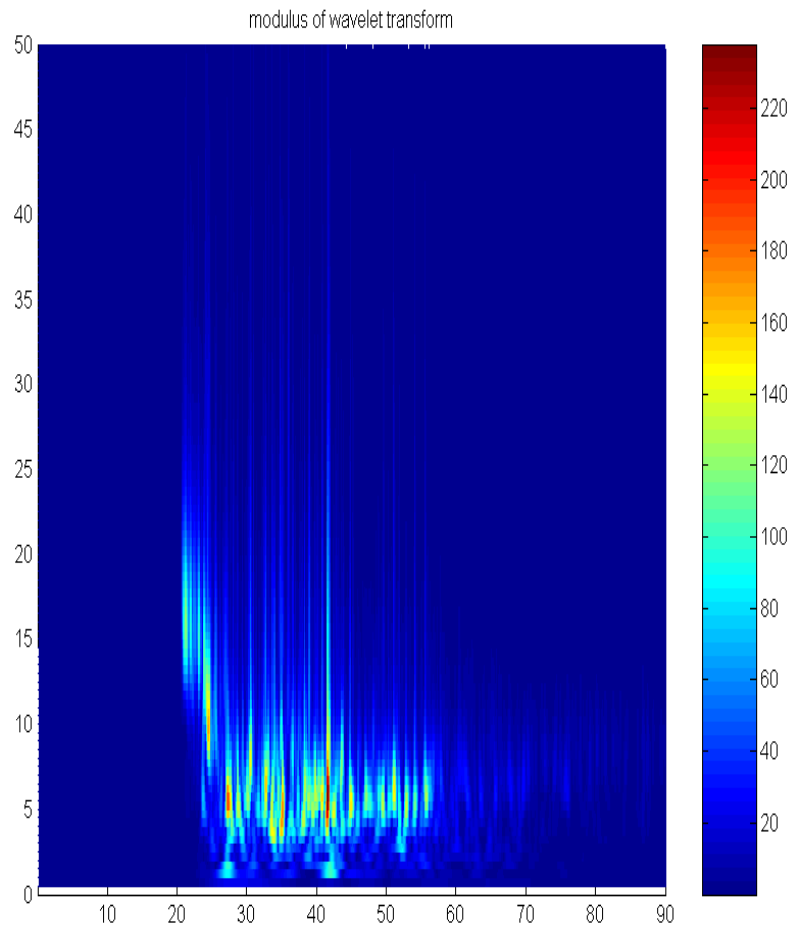
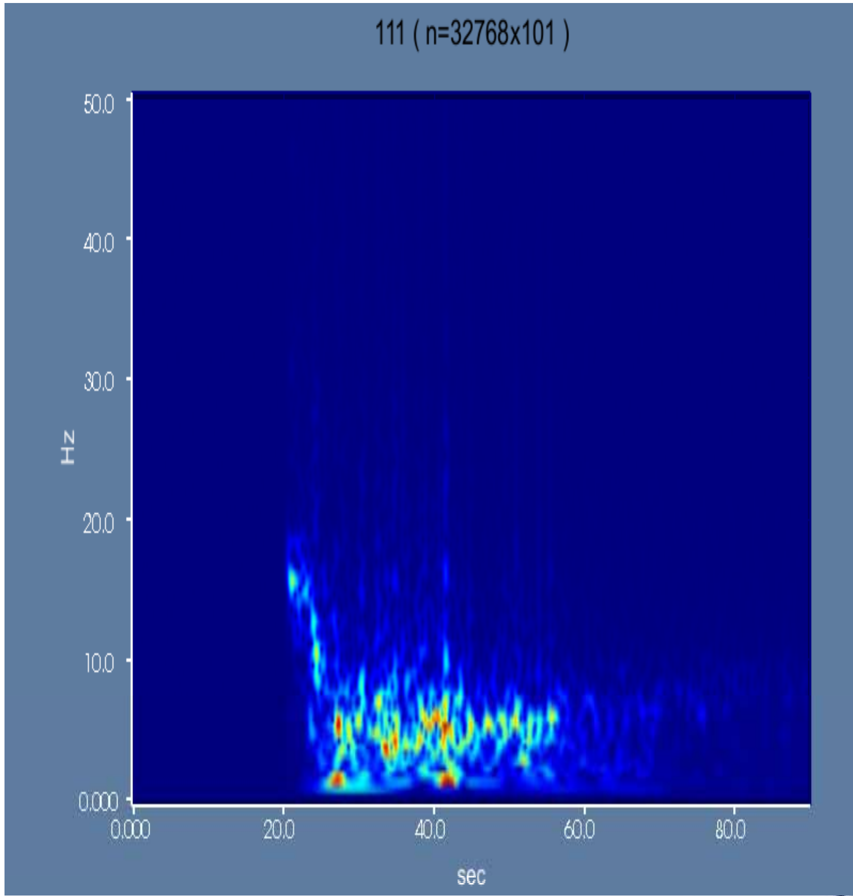
Chi-Chi (921) Earthquake

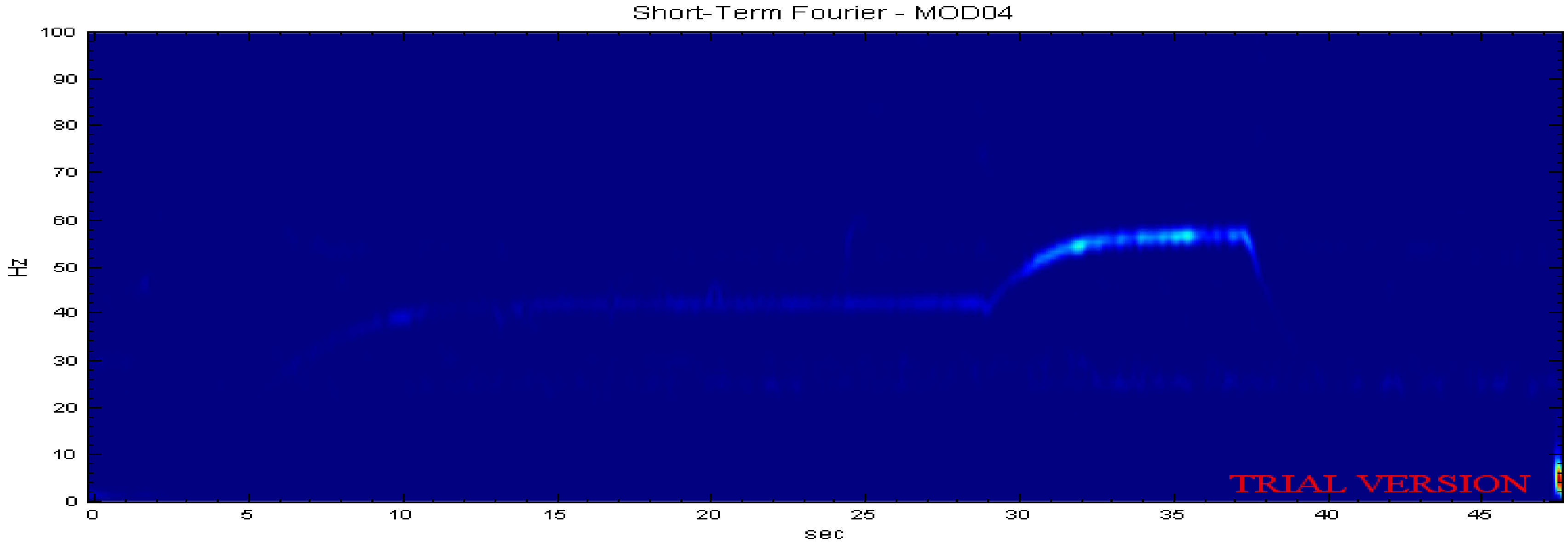


Morlet Transform by MATLAB

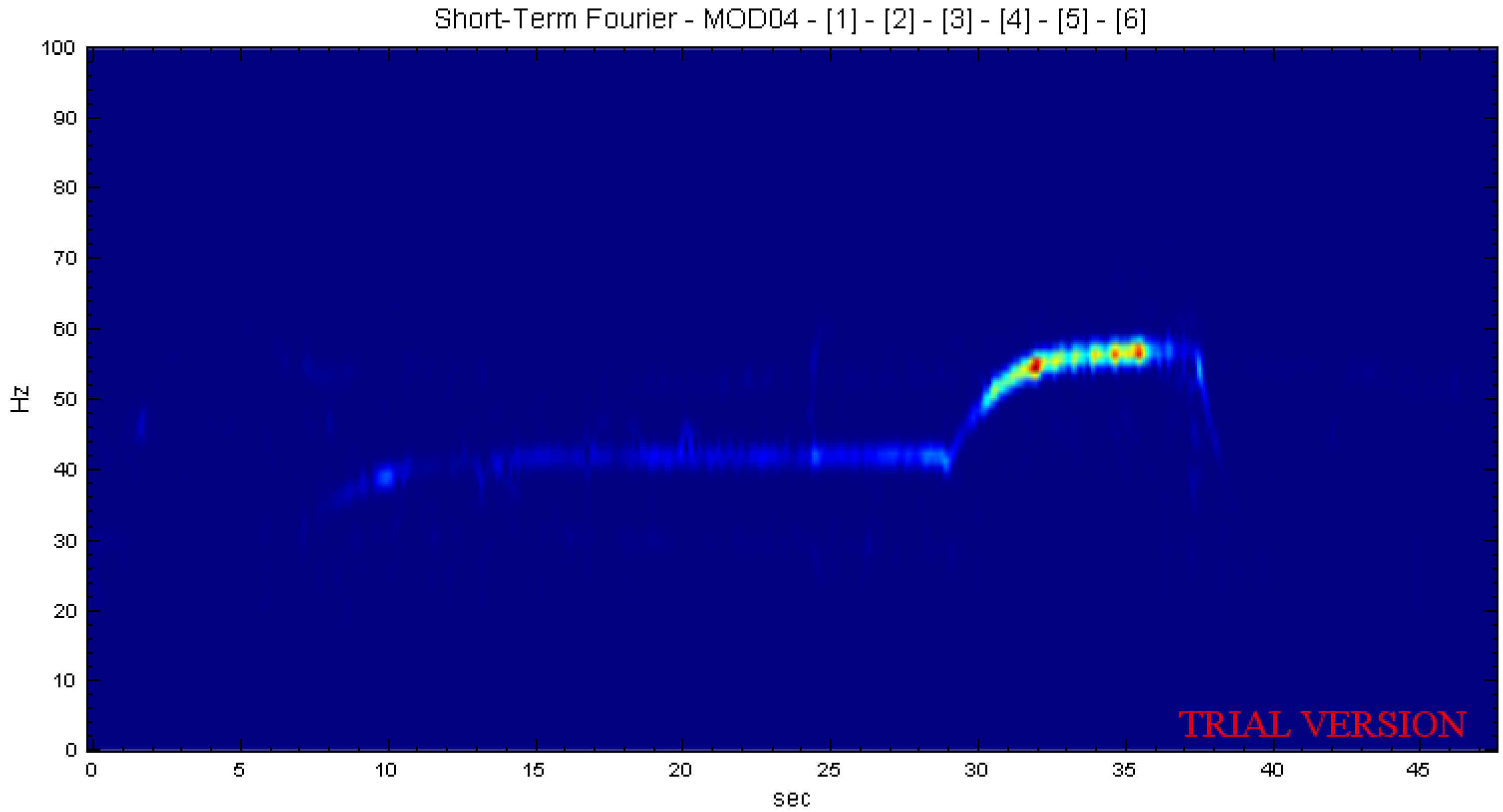


Morlet-Jeng Transform

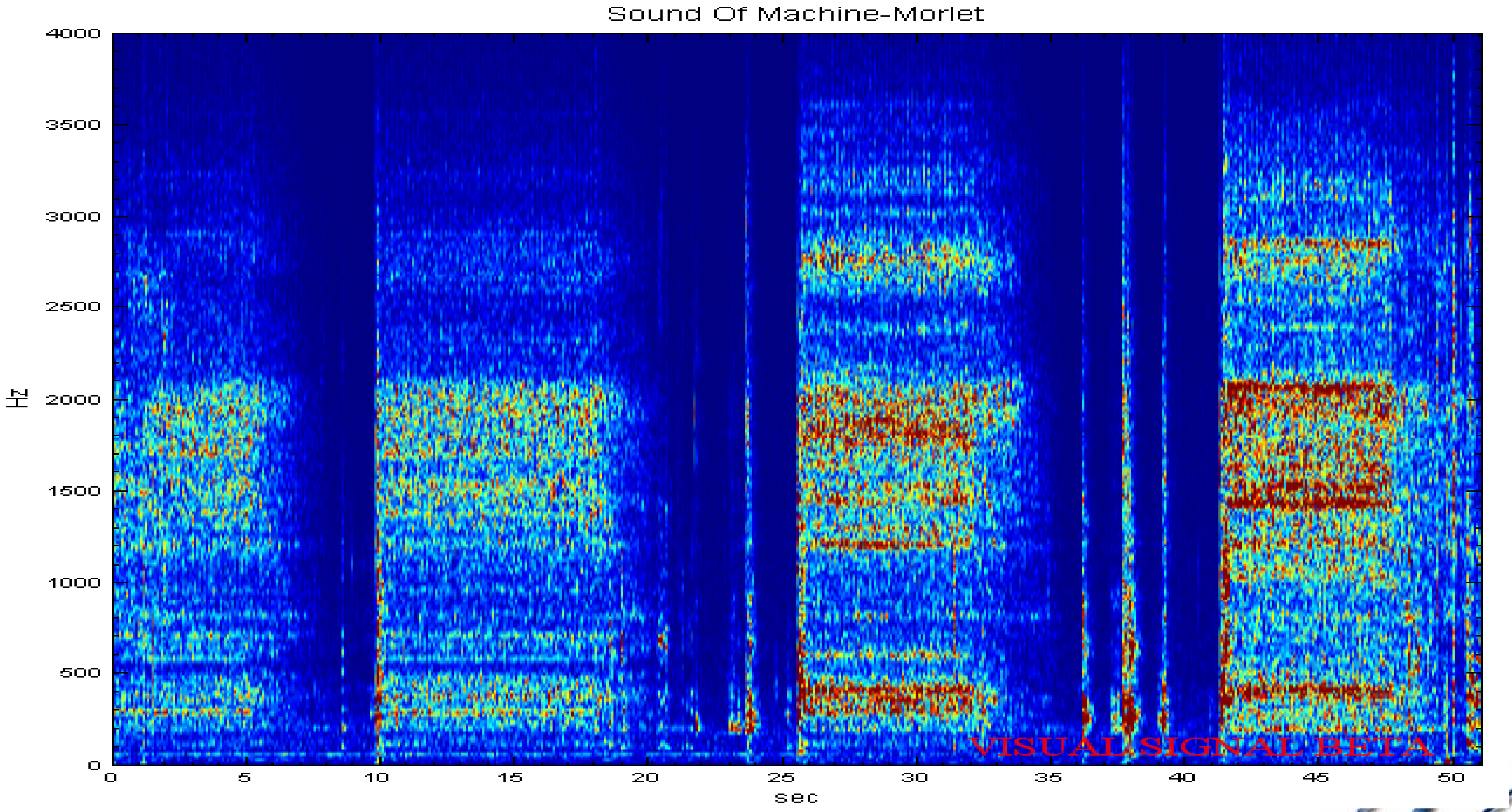
Sound of Fan


Short-Term Fourier - MOD04
100
90
80
70
60
50
40
30
20
10
0
Hz
0
5
10
15
20
25
30
35
40
45
sec
TRIAL VERSION

Sound of Fan (IMF6)



Sound of Machine



Instantaneous Frequency and Hilbert Transform

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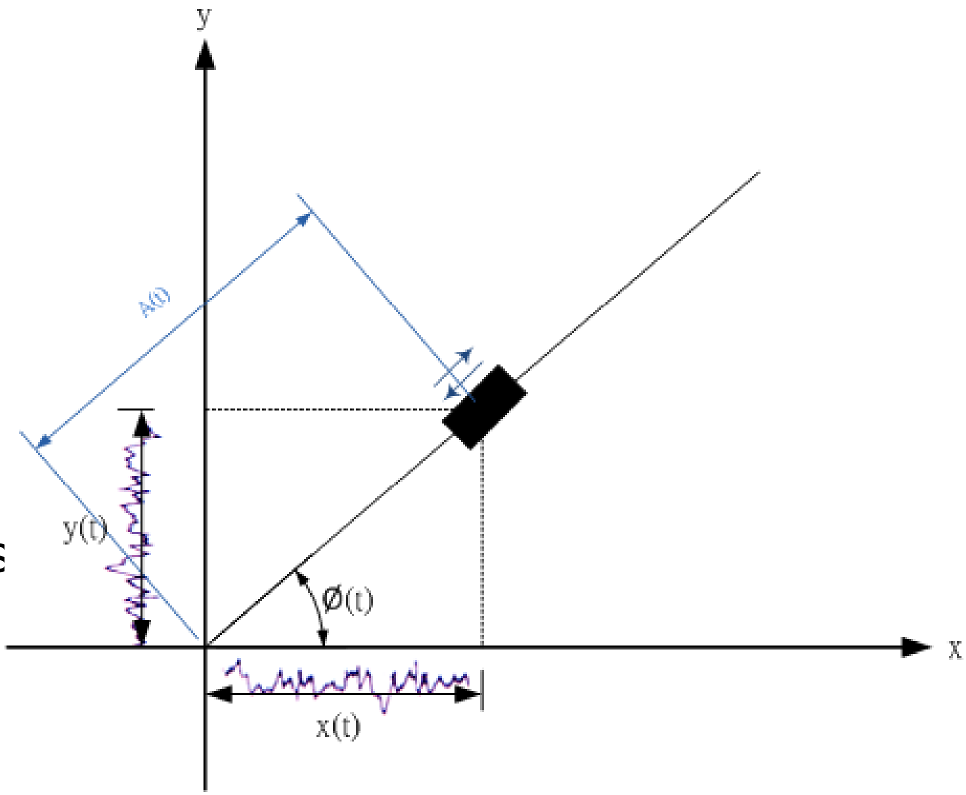
Instantaneous Frequency

- A time-series signal can be regarded as the x-axis projection of a planar slider motion. The slider moves along a rotating stick with axial velocity $\dot{a}(t)$, while the stick is spinning with an angular speed $\dot{\varphi}(t)$.
- The corresponding y-axis projection shares the same frequency distribution as x-axis signal with 90 degree phase shift. Such conjugate signal is evaluated from Hilbert Transform.



Hilbert Transform

$$y(t) = \frac{1}{\pi} PV \int \frac{x(\tau)}{t-\tau} d\tau$$

$$z(t) = x(t) + iy(t) = a(t)e^{i\varphi(t)}$$

$$a(t) = \sqrt{x^2 + y^2}$$

$$\varphi(t) = \tan^{-1}\frac{y(t)}{x(t)}$$

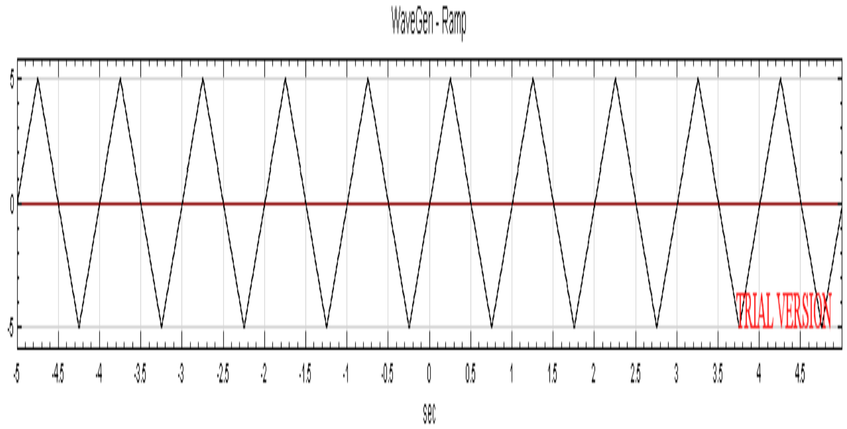
$$z(t) = \frac{2}{\sqrt{2\pi}} \int_0^\infty S(\varpi)e^{iwt} d\varpi$$

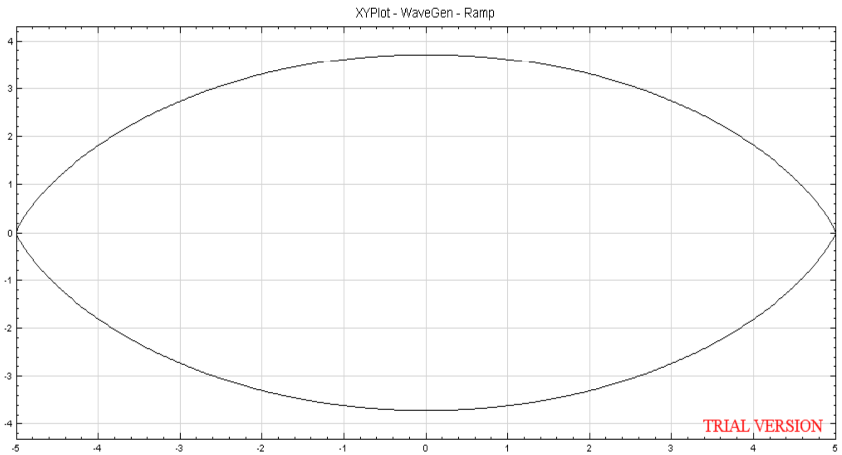
where

$$S(\varpi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty x(t)e^{-iwt} dt$$

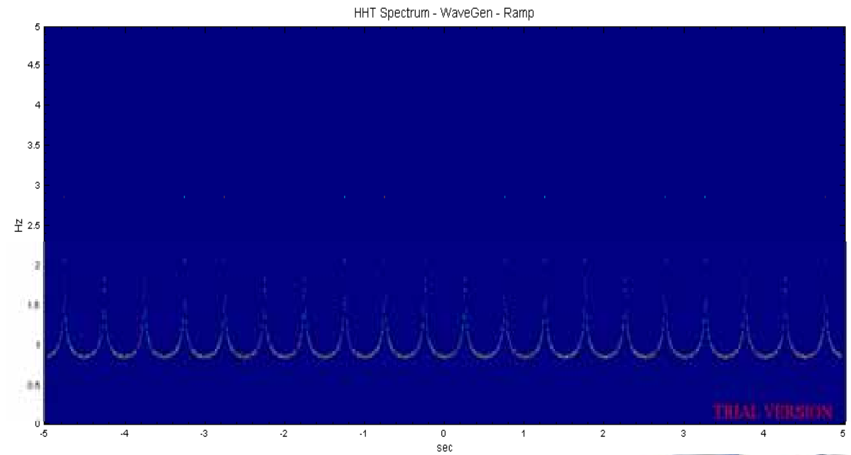
Instantaneous frequency: $\omega = -\frac{\partial\varphi}{\partial t}$

Hilbert transform of triangular signal





Phase diagram



Hilbert spectrum

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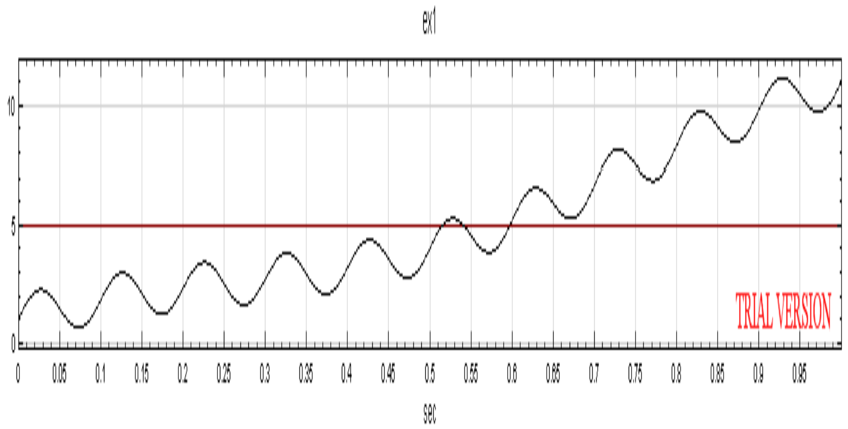
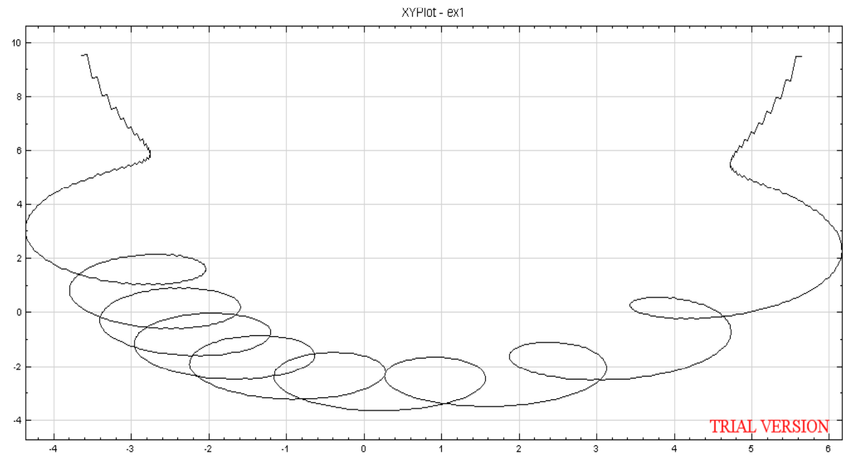
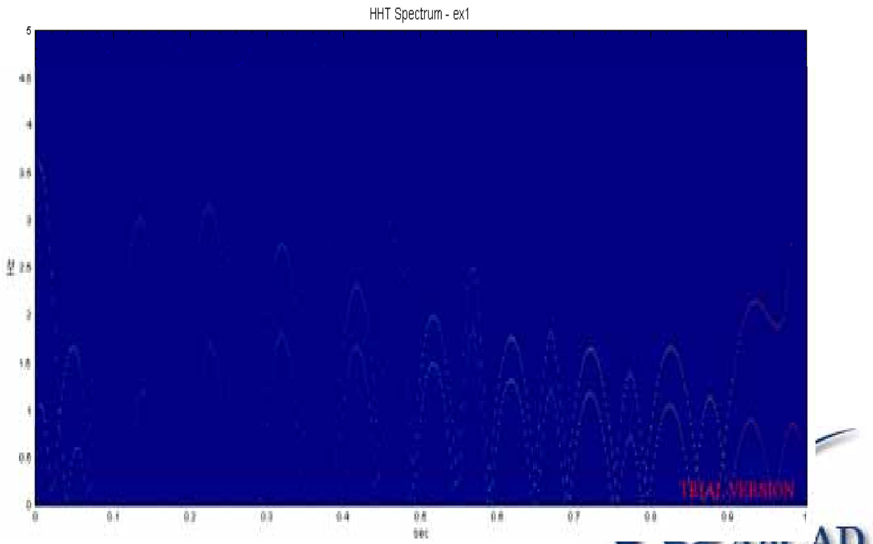
Drawback of Hilbert Transform

- Negative instantaneous frequency occurs for signal not having equal number of extreme and zero-crossing points.
- Too much DC offset also results in negative frequency.







Intrinsic Mode Function (IMF)

- Definition: “Any function having the same numbers of zero-crossings and extrema, and also having symmetric envelopes defined by local maxima and minima respectively is defined as an Intrinsic Mode Function (IMF). “ ~ Norden E. Huang
- An IMF enjoys good properties of Hilbert transform.
- A signal can be regarded as composition of several IMFs. IMF can be obtained through a process called Empirical Mode Decomposition. (ref. Dr. Hsieh’s presentation file)

The advantages of using HHT

- Removal of non-periodical part
- Separation of carrier frequency: even though the spectrum is close. Such function can be hardly achieved by frequency based filter.
- Nonlinear effect might introduce frequency harmonics in spectrum domain. Through HHT, the nonlinear effect can be caught by EMD/IMF. The marginal frequency therefore enjoys shorter band width.
- Average frequency in each IMF represents intrinsic signature of physics behind the data.
- Signal can be regarded as generated from rotors of different rotating speeds (analytical signals).

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Time-Frequency Analysis Comparison

	Fourier Transform	STFT	Morlet / Enhanced Morlet	Hilbert Transform	HHT
Instantaneous frequency	n/a	distribution	distribution	Single value	Discrete values
Frequency change with time	no	yes	yes	yes	yes
Frequency resolution	good	ok	ok/good	good	good
Adaptive base	no	no	no	n/a	yes
Handling non-linear effect	n/a	no	no	yes	yes

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Geo-Science Applications

Tidal wave

Tsunami

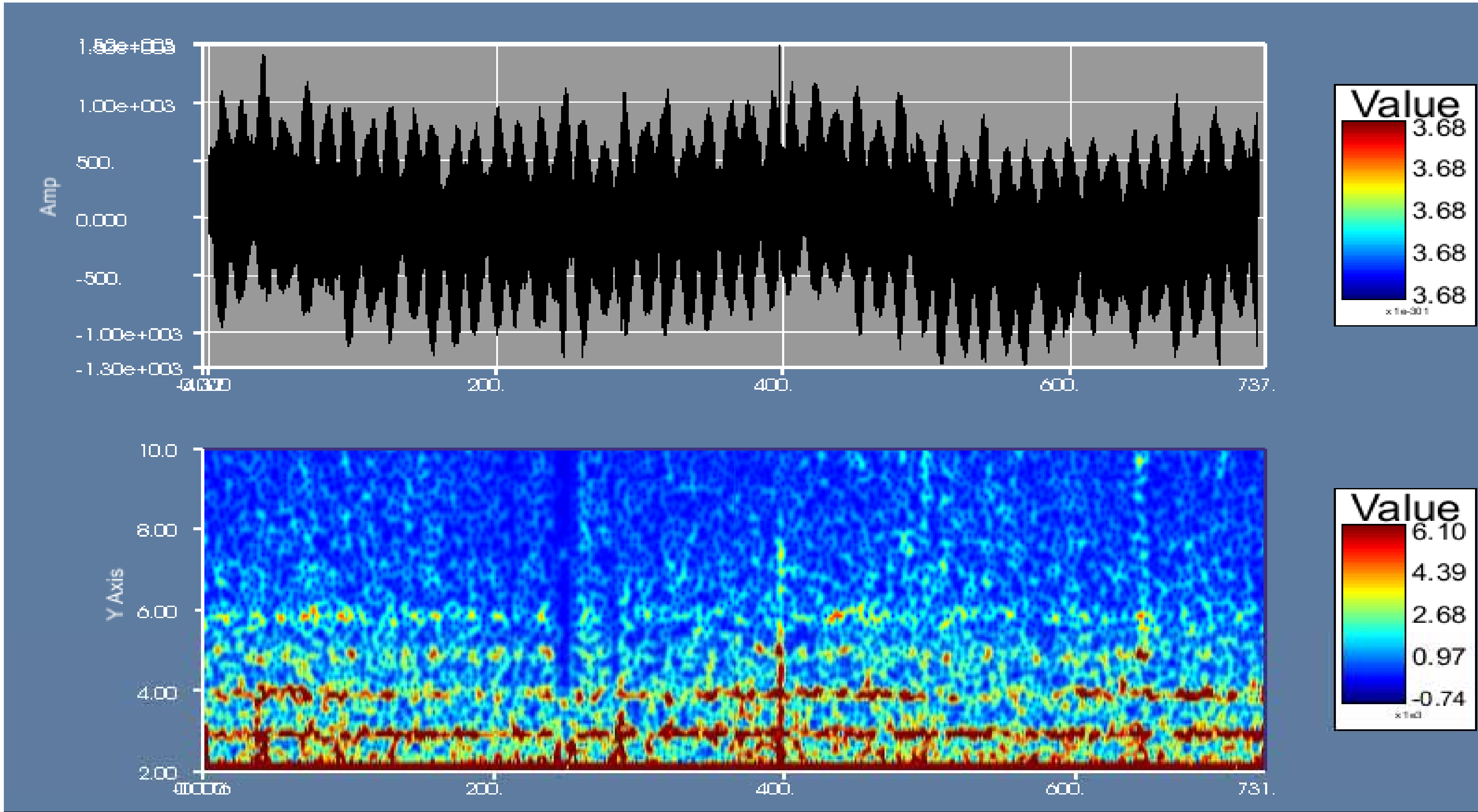
Tidal waves

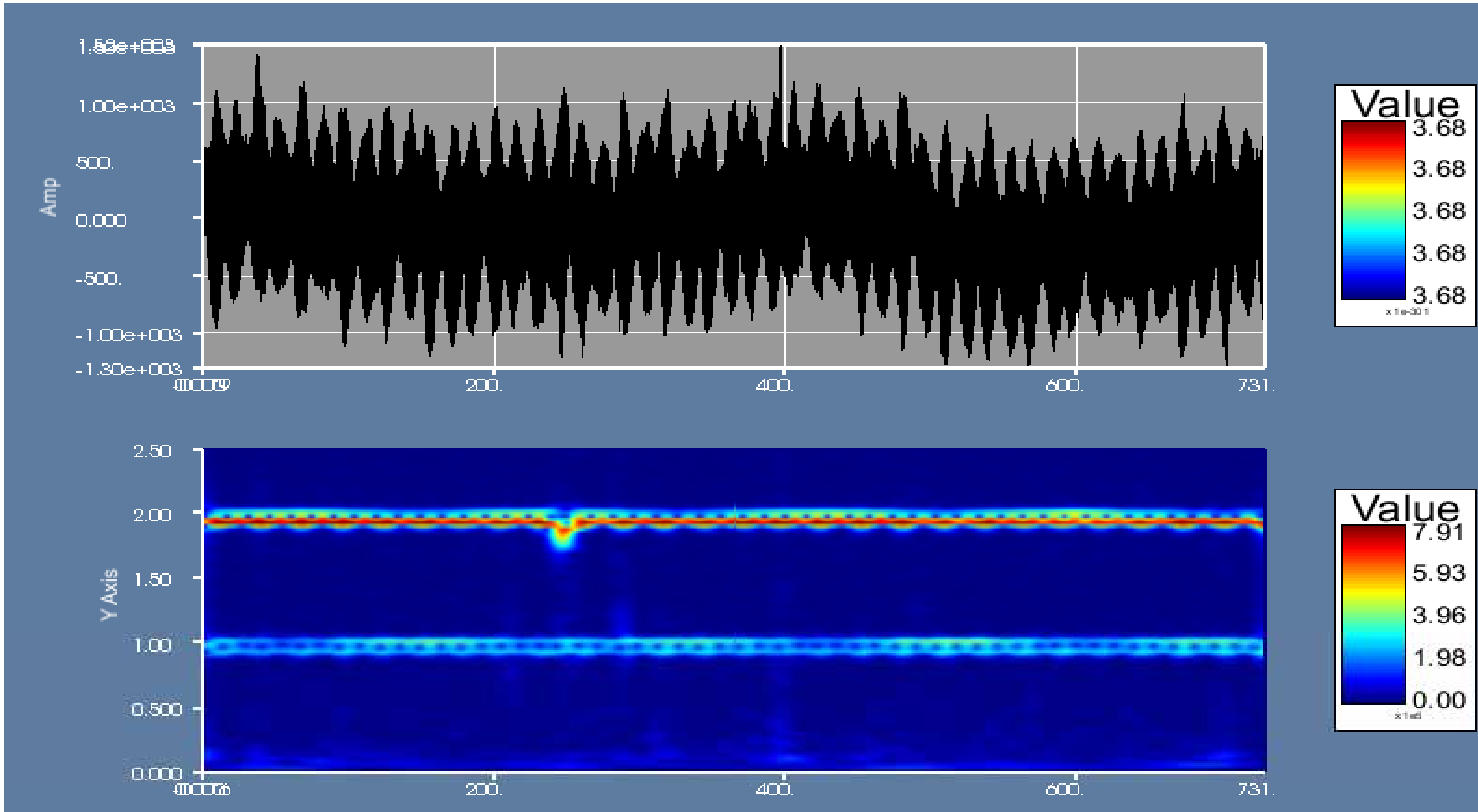
Time unit: day

Recorded period: 2003/11/01:00 to 2003/12/31:23

Sampling: every 6 min.

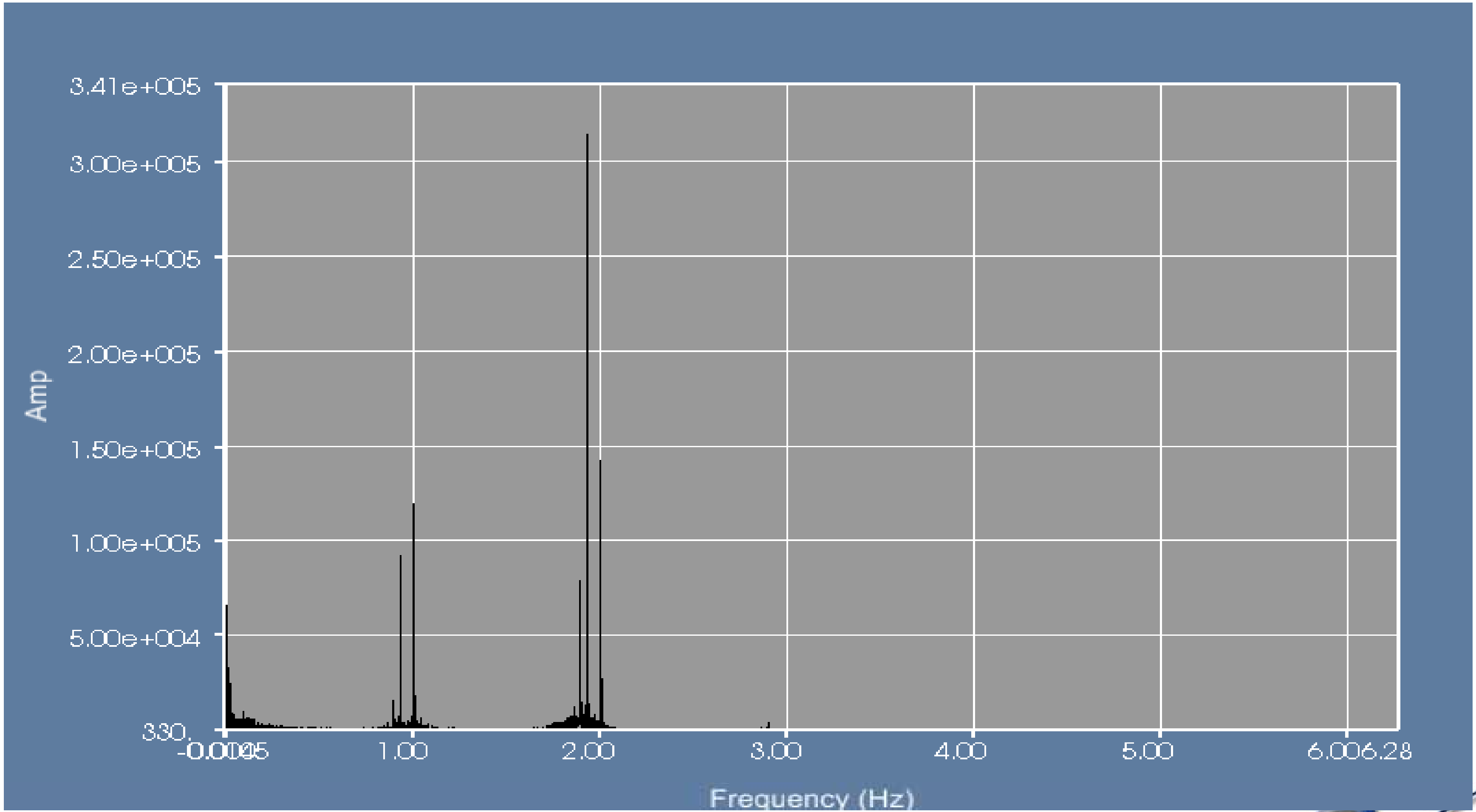
Tide signal (2 years)





Amp
1.00e+003
500.
0.000
-500.
-1.00e+003
-1.30e+003
200.
400.
600.
731.
Value
3.68
3.68
3.68
3.68
3.68
Y Axis
2.50
2.00
1.50
1.00
0.500
0.000
200.
400.
600.
731.
Value
7.91
5.93
3.96
1.98
0.00

Tide signal - Spectrum (2 years)

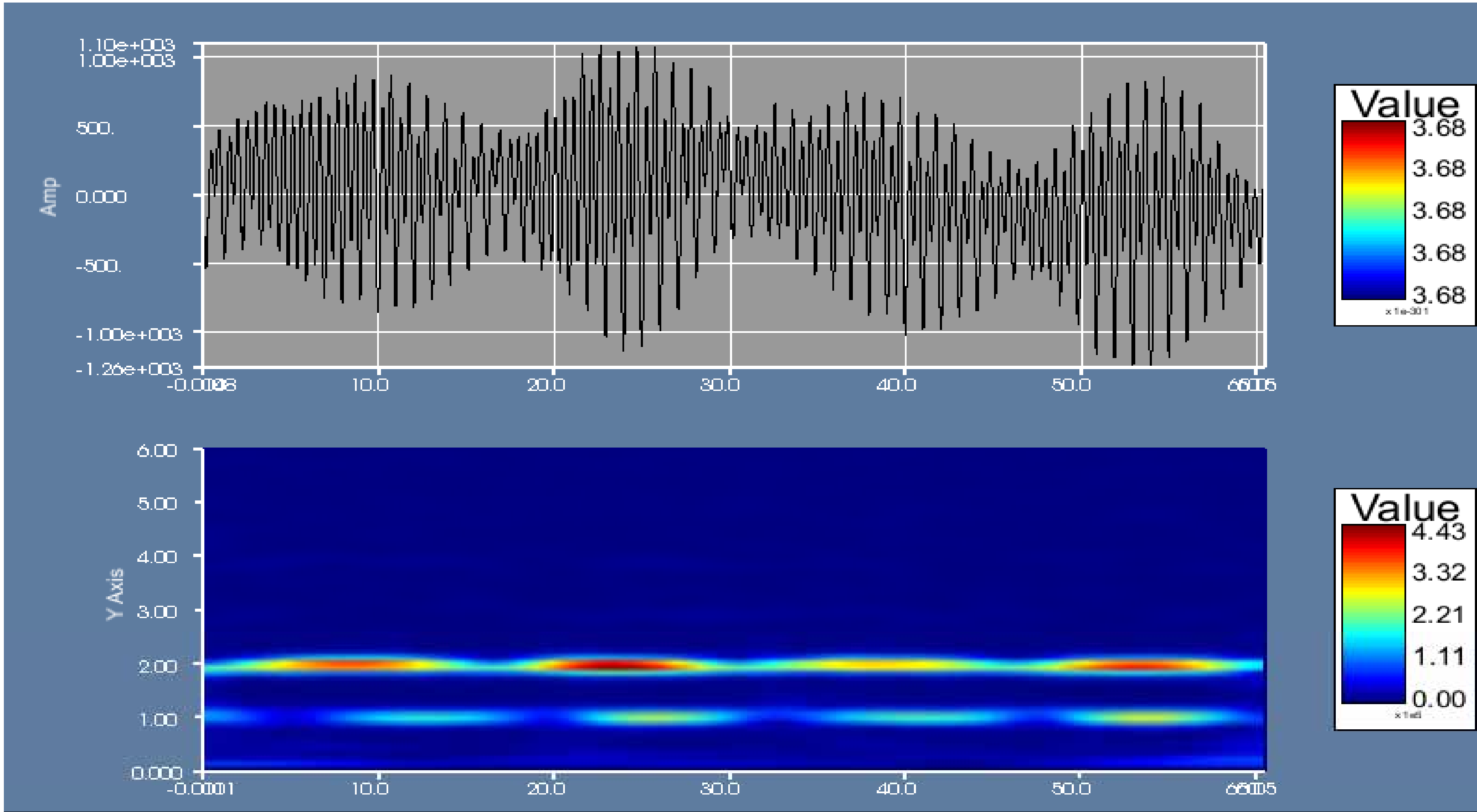


Tsunami

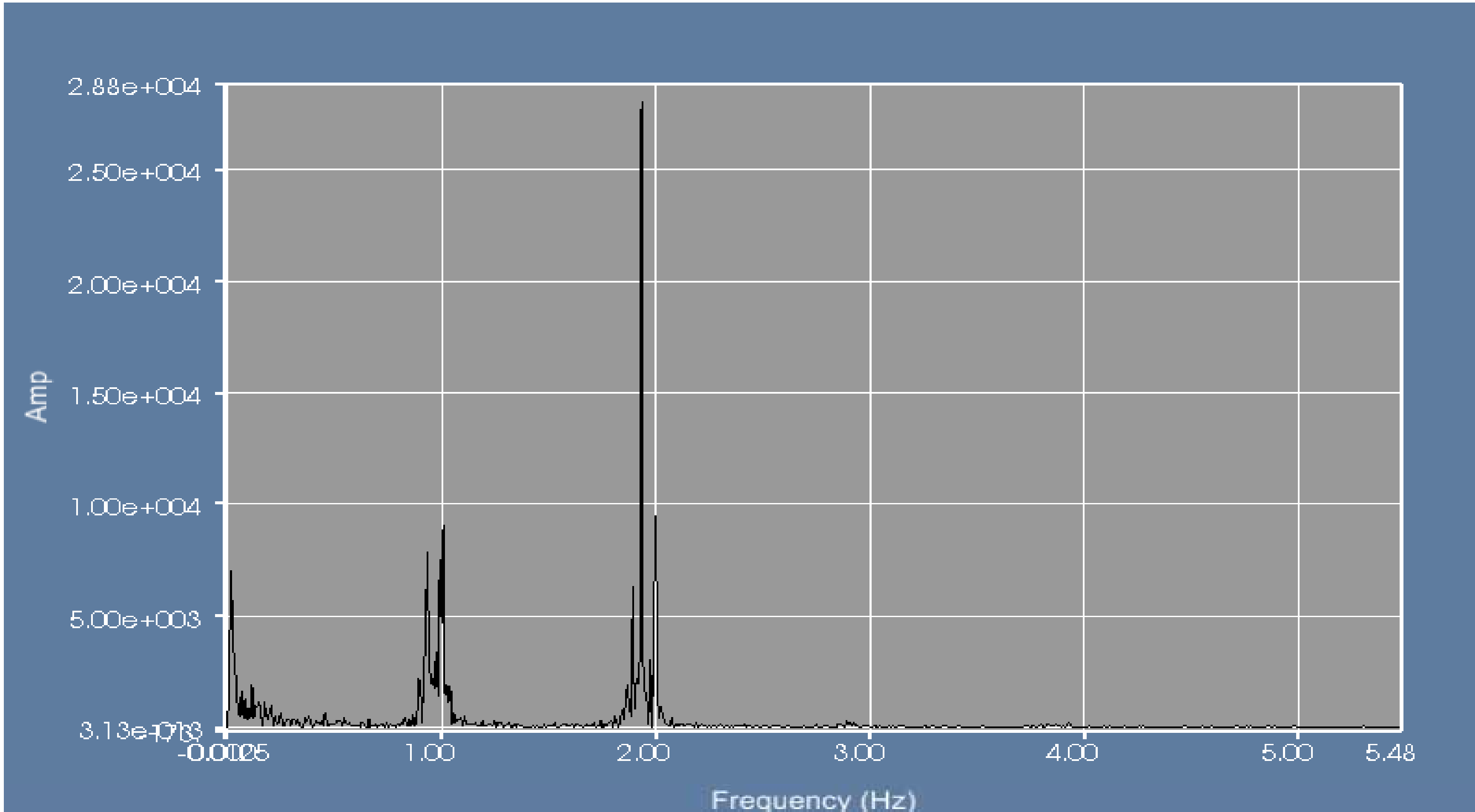
Time unit: day

Recorded period: 2003/11/01:00 to 2003/12/31:23

Low frequency analysis

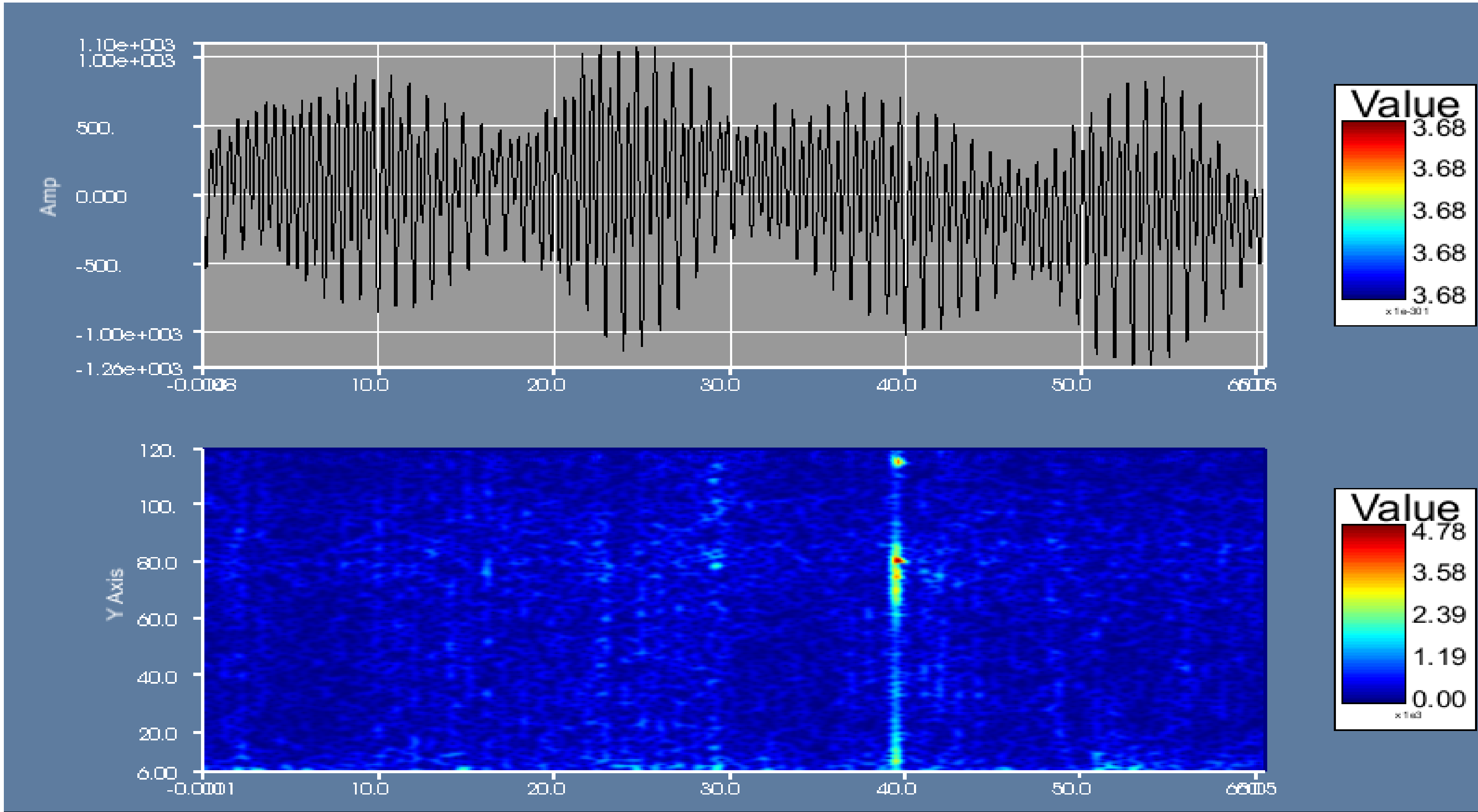


Spectrum

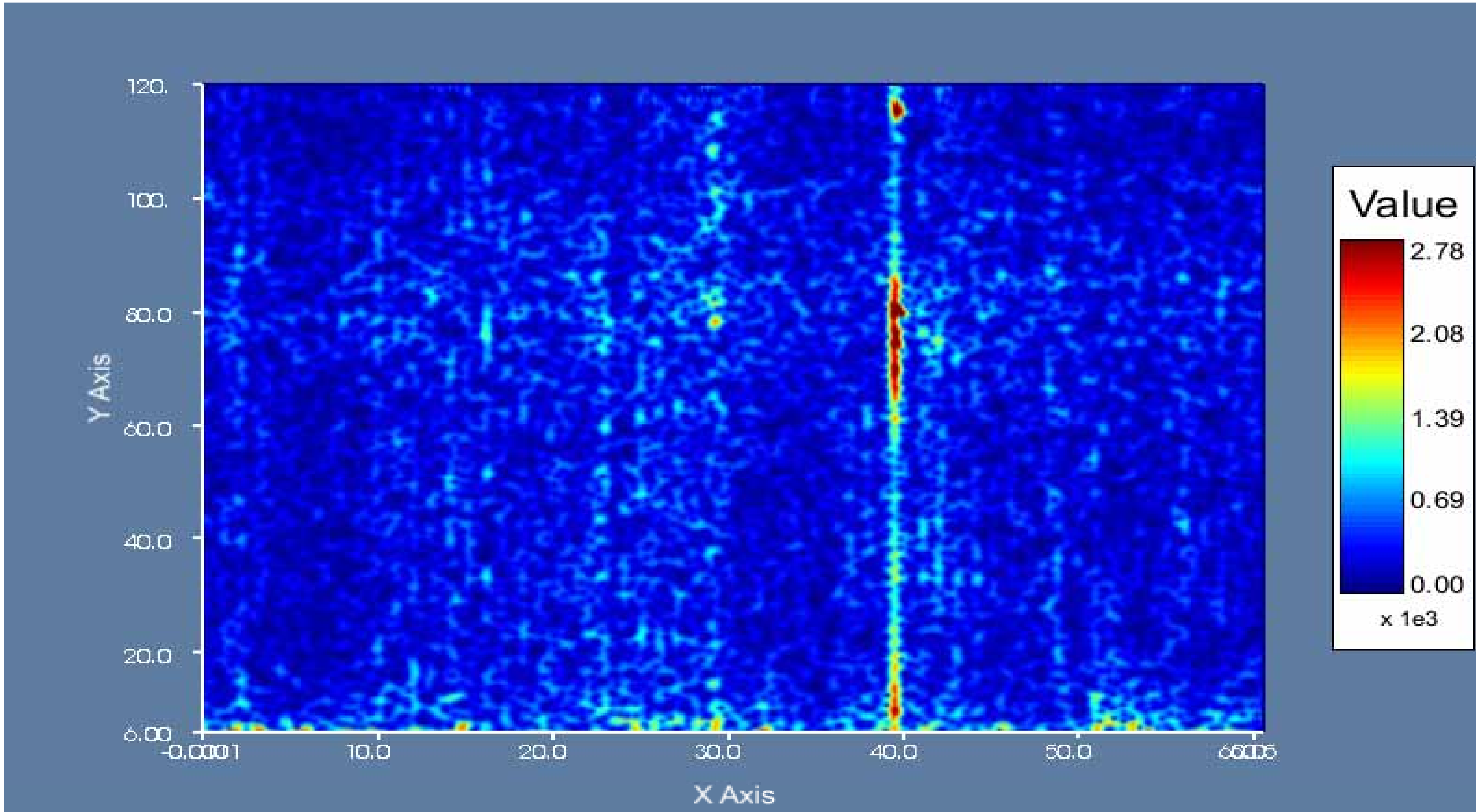


2.88e+004
2.50e+004
2.00e+004
1.50e+004
1.00e+004
5.00e+003
Amp
1.00
2.00
3.00
4.00
5.00
5.48
Frequency (Hz)

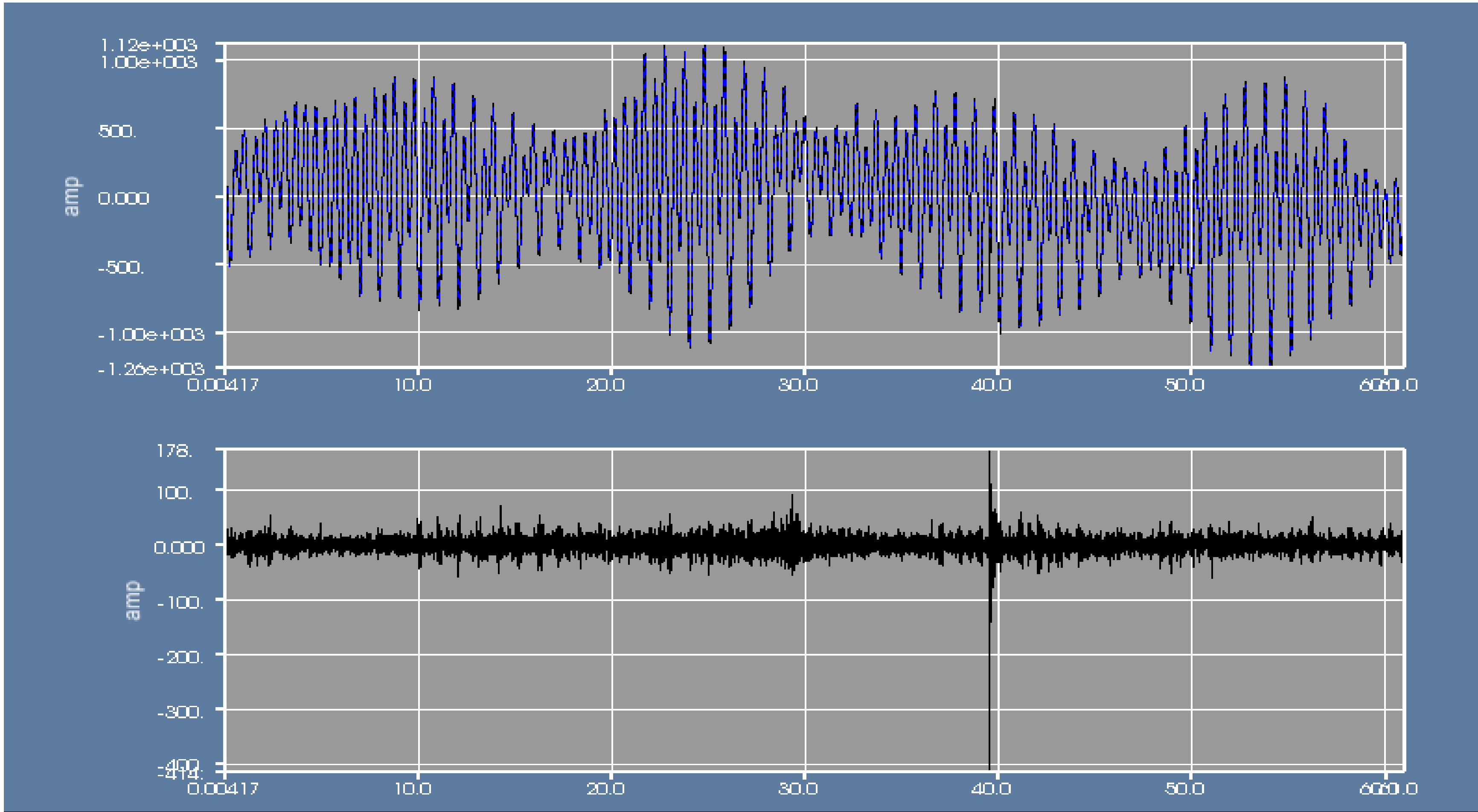
Freq. : 6 to 120 (1/day)



Freq. : 6 to 120 (1/day)



Apply Gaussian filter (fH=6, fL=fH/1.5)



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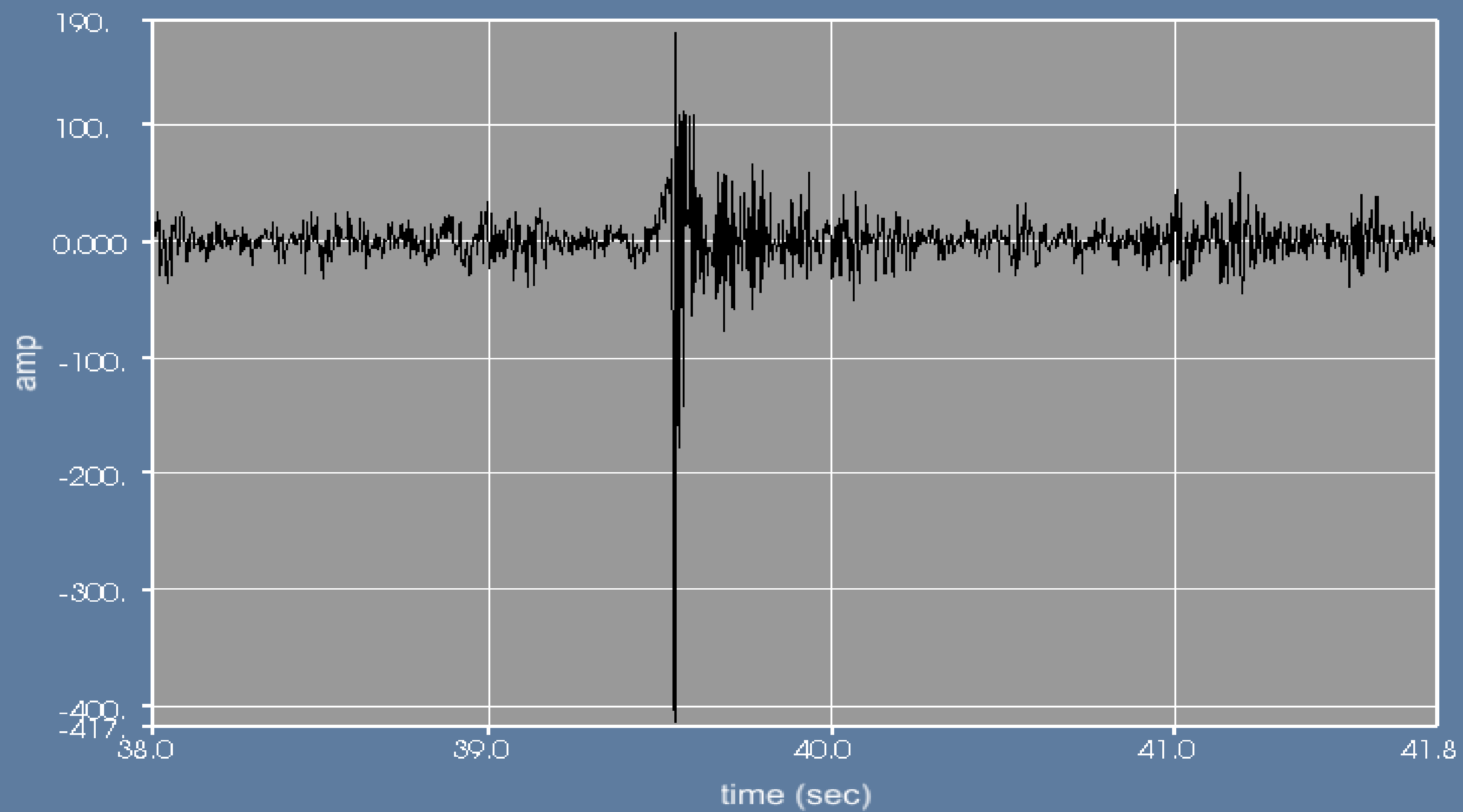
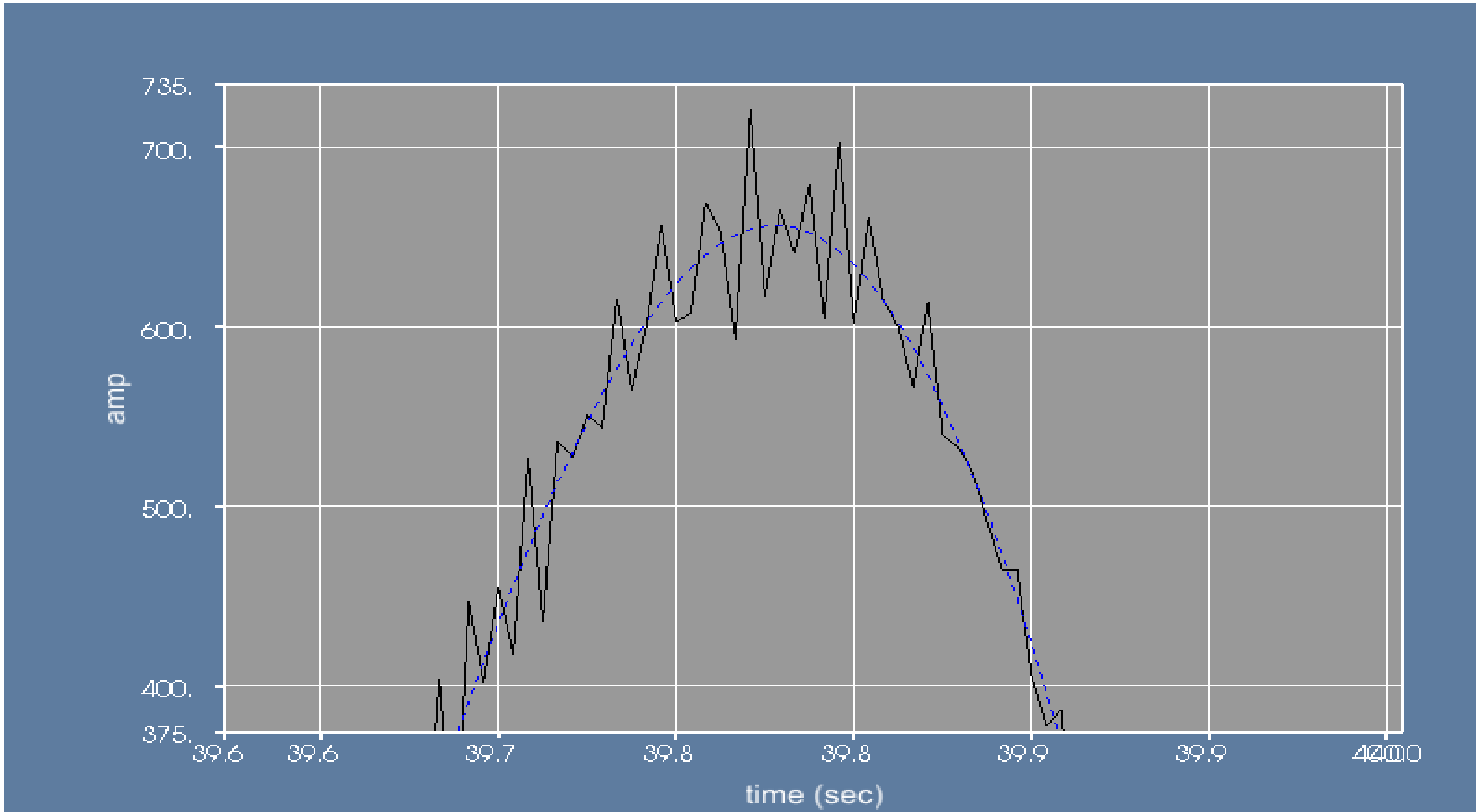
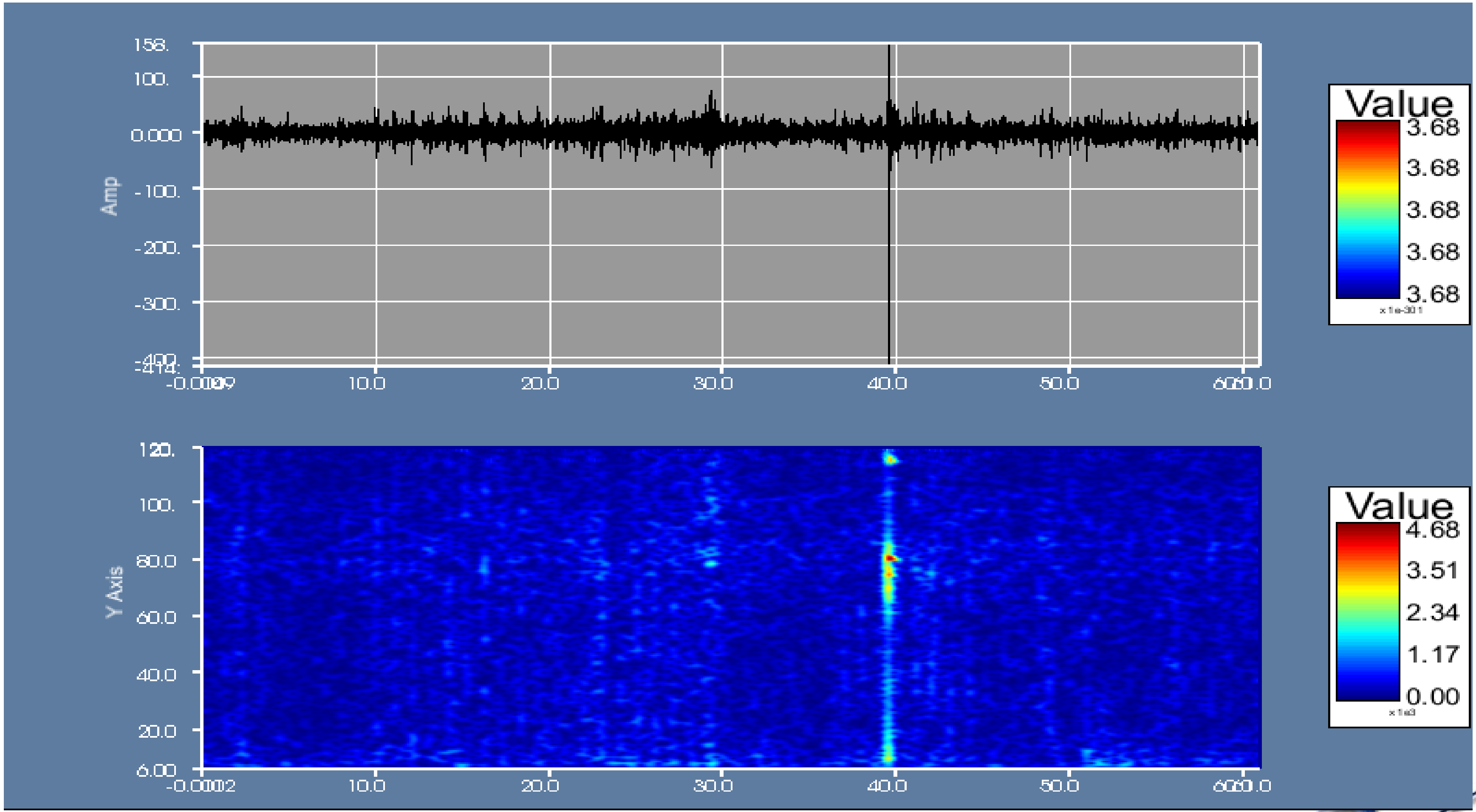


190.
100.
0.000
-100.
-200.
-300.
-400.
-417.
amp
38.0
39.0
40.0
41.0
41.8
time (sec)

Original vs. filtered signals



Time-Frequency plot of filtered signal



Earth Quake Signal

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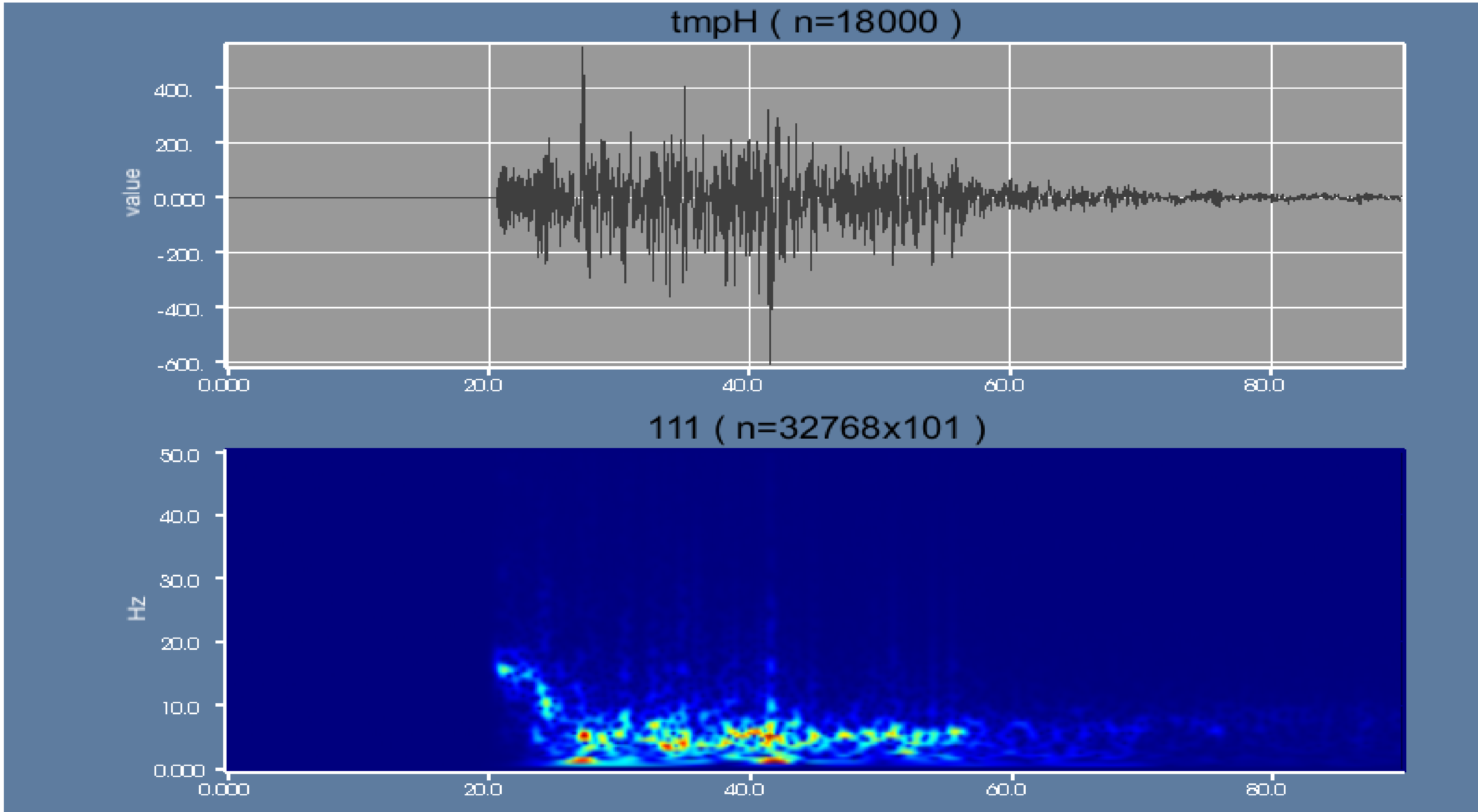
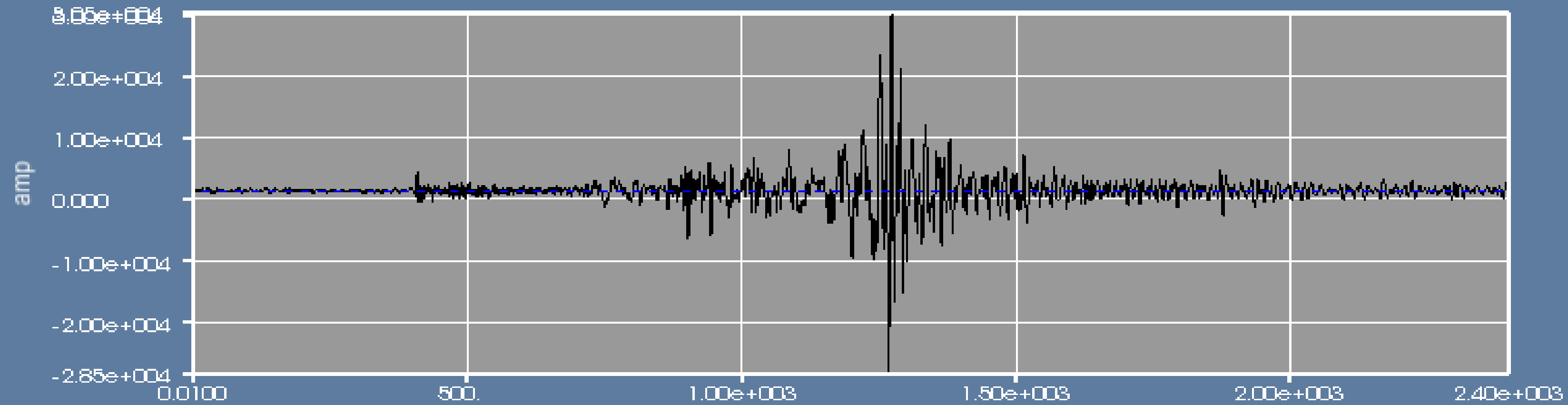
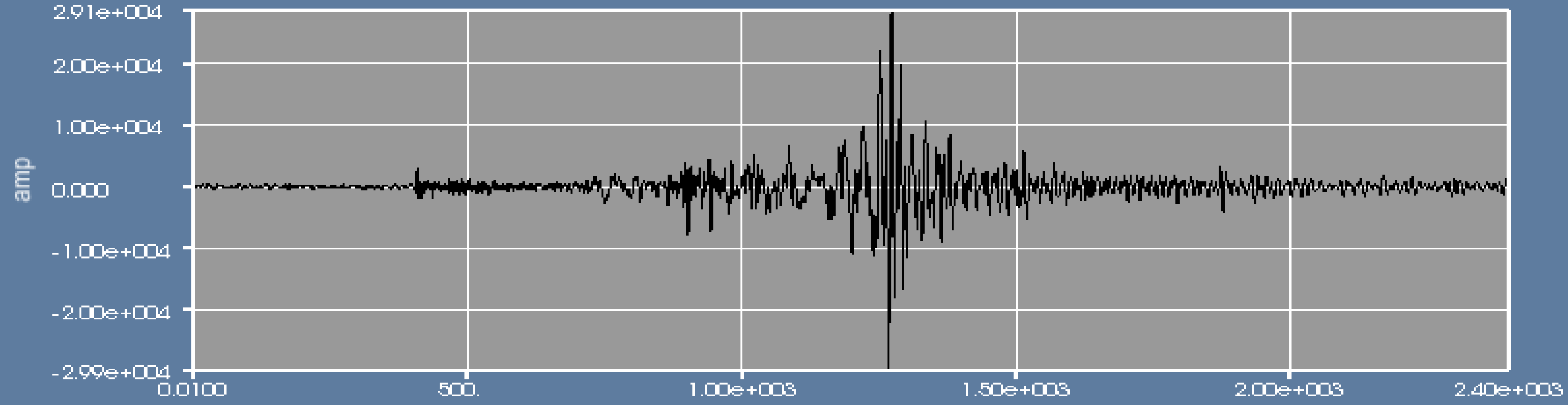
Chi-Chi (921) Earthquake

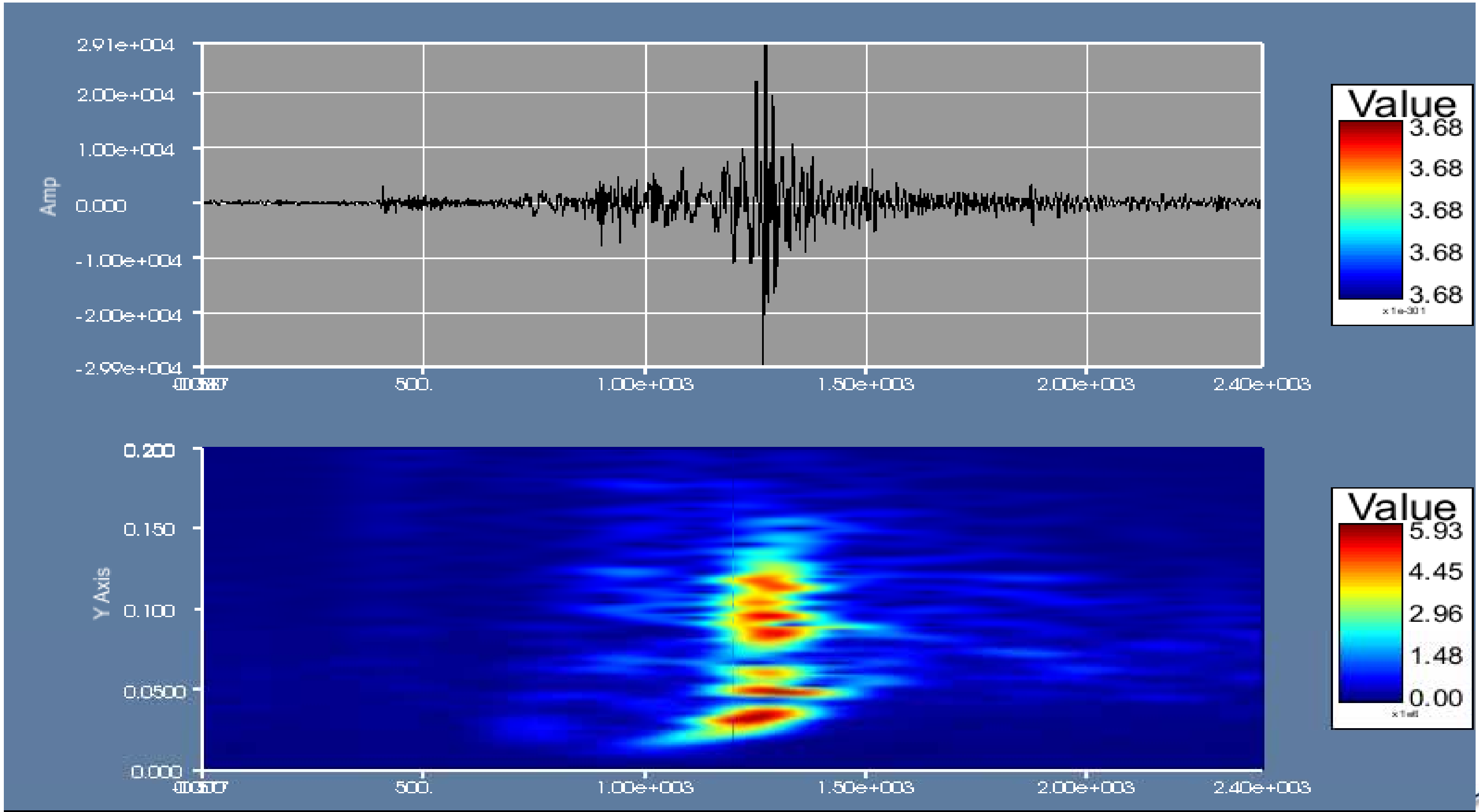


Earthquake (YHNB_V)

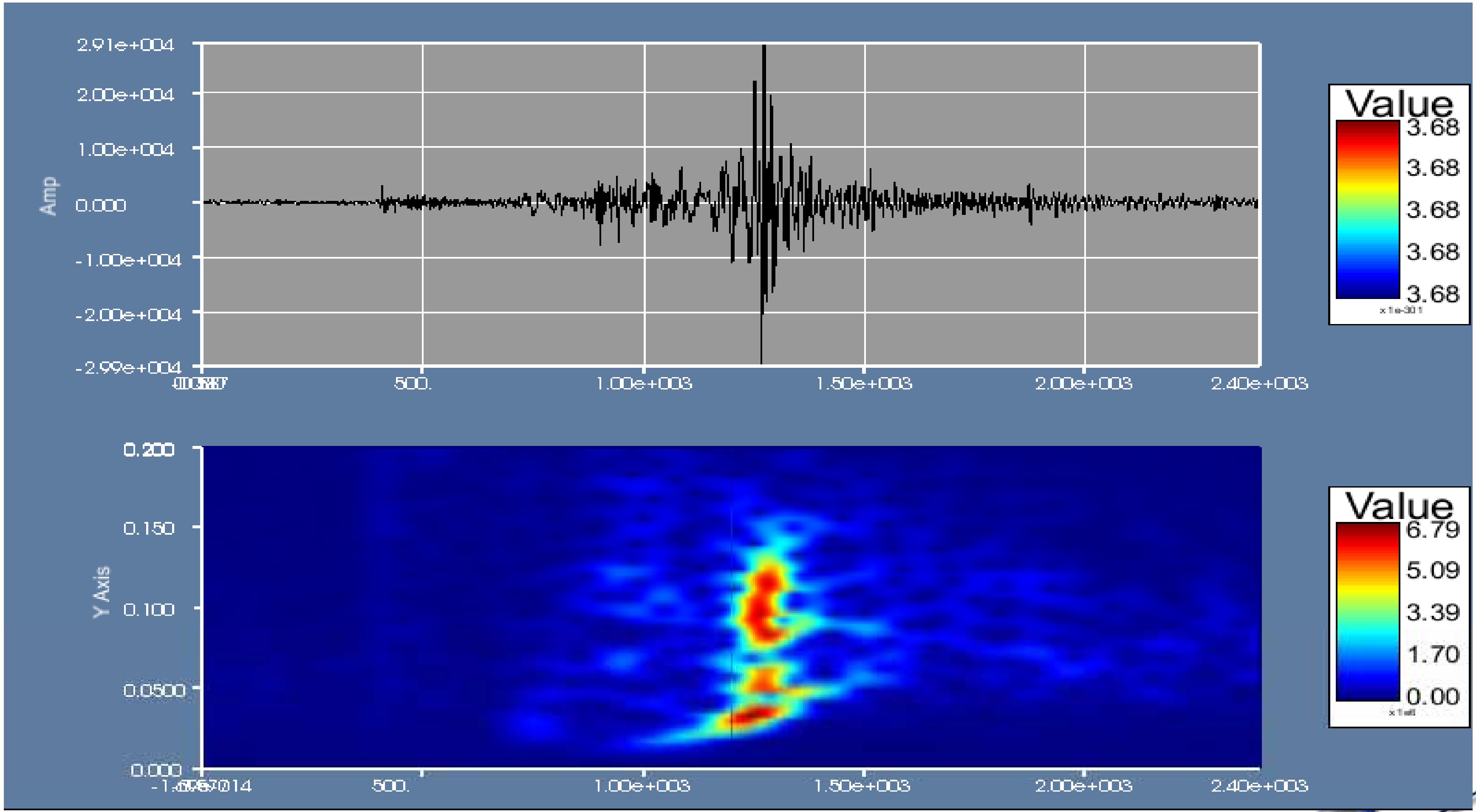




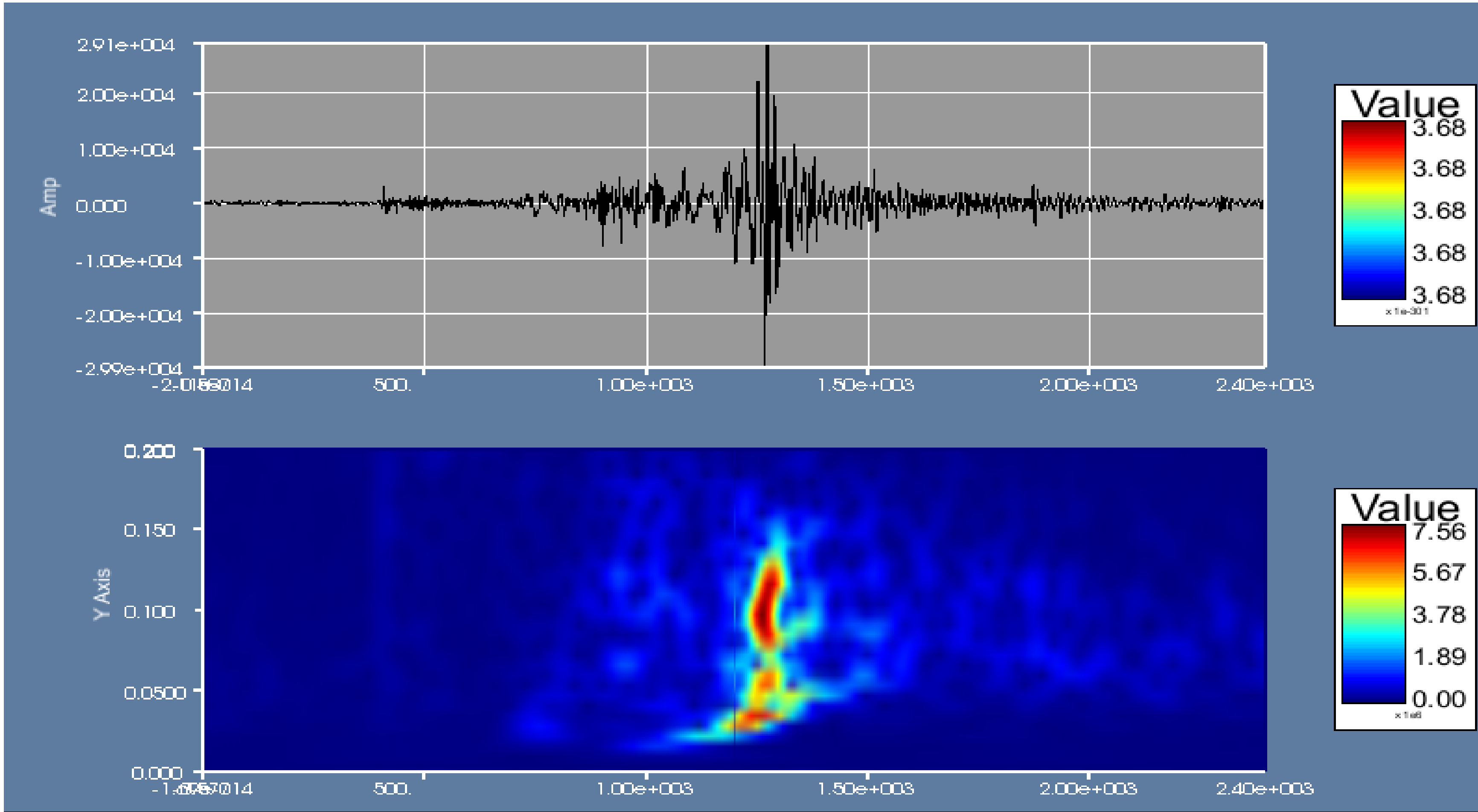
Uncertainty Principle (nf=100)



Uncertainty Principle (nf=50)



Uncertainty Principle (nf=30)



Thank you!